Linear Filters

- **Linear filtering:**
  - Form a new image whose pixels are a weighted sum of original pixel values, using the same set of weights at each point

Convolution

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

- Represent the linear weights as an image, ***F***
- ***F*** is called the **kernel**
- Operation is called **convolution**
  - Center origin of the kernel F at each pixel location
  - Multiply weights by corresponding pixels
  - Set resulting value for each pixel

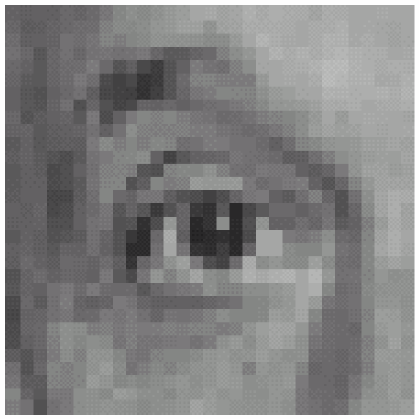
- Image, ***R***, resulting from convolution of ***F*** with image ***H***, where u,v range over kernel pixels:

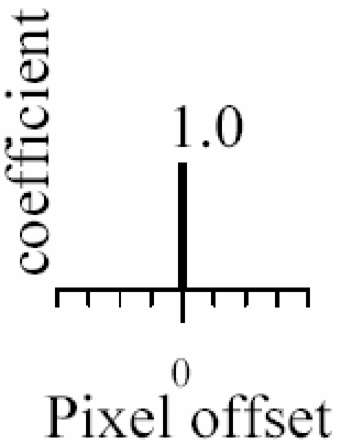
$$R_{ij} = \sum_{u,v} H_{i-u,j-v} F_{uv}$$

Warning: the textbook mixes up H and F

Slide credit: David Lowe (UBC)

Linear filtering (warm-up slide)

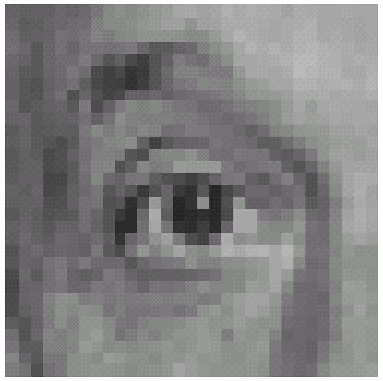

original



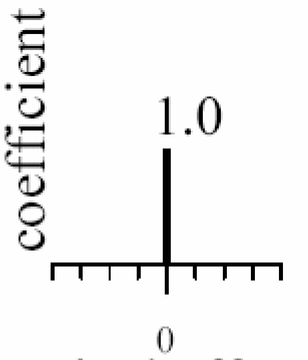
?

Slide credits for these examples: Bill Freeman, David Jacobs

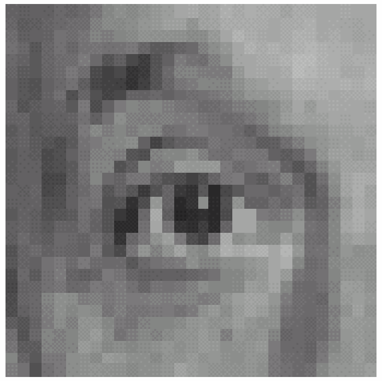
Linear filtering (warm-up slide)



original

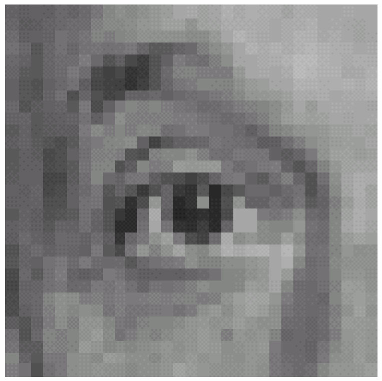


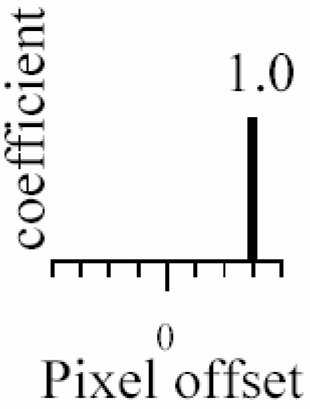
Pixel offset



Filtered
(no change)

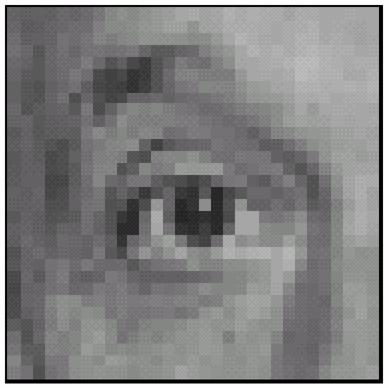
Linear filtering

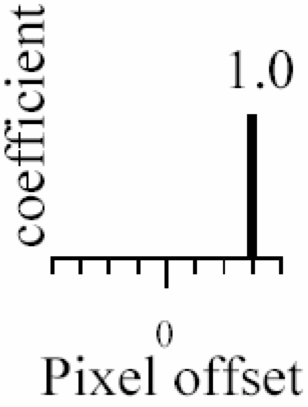

original

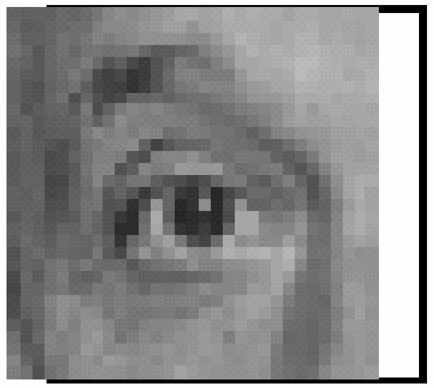


?

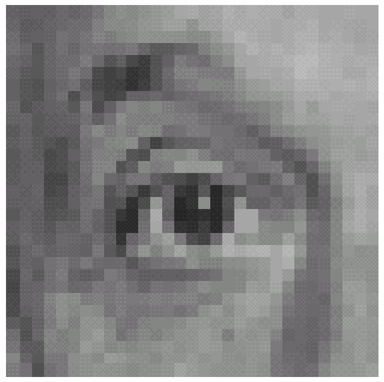
shift


original

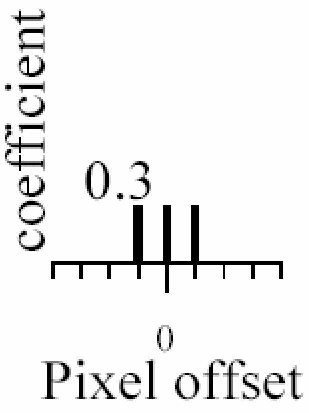



shifted

Linear filtering

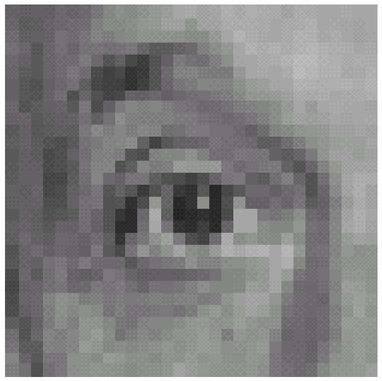


original

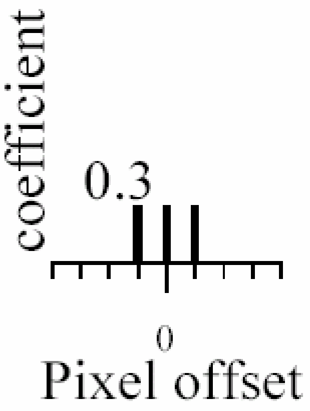


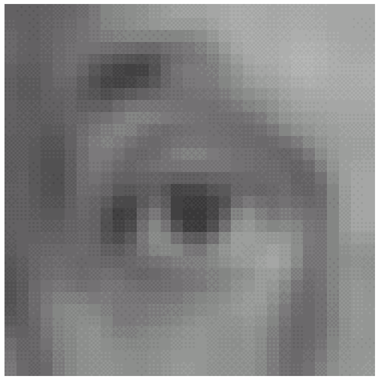
?

Blurring



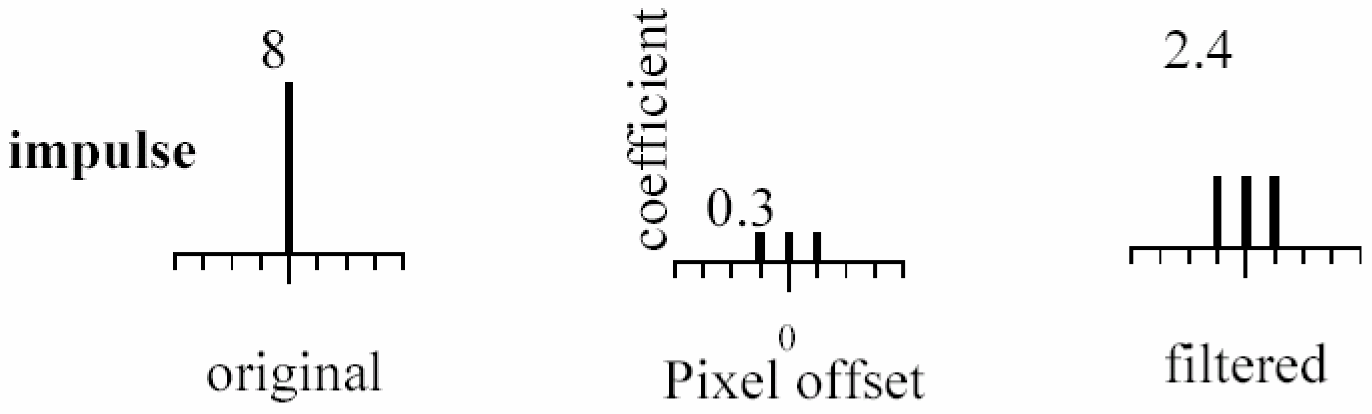
original



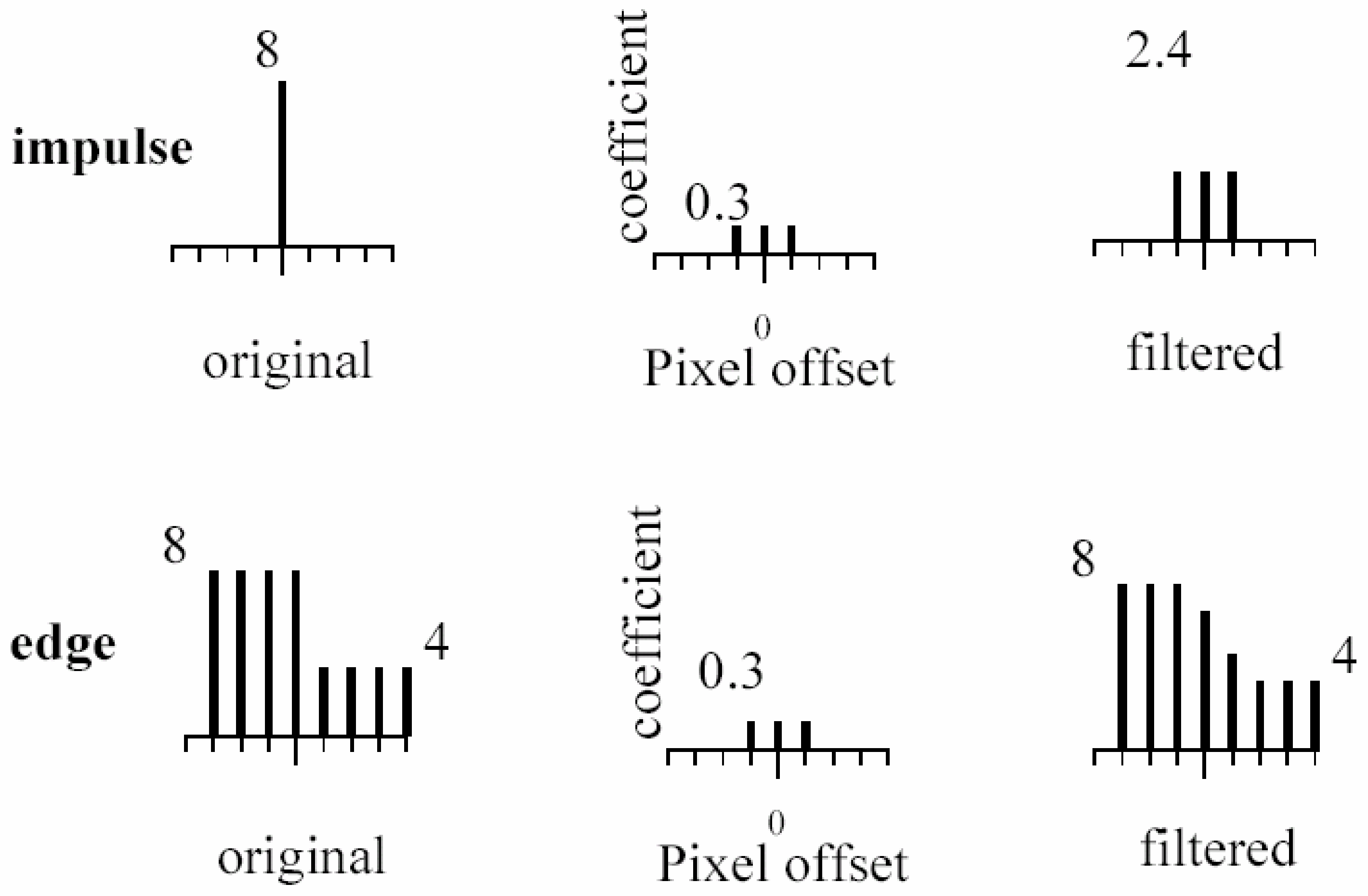


Blurred (filter applied in both dimensions).

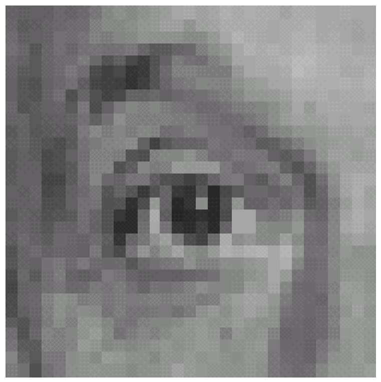
Blur examples



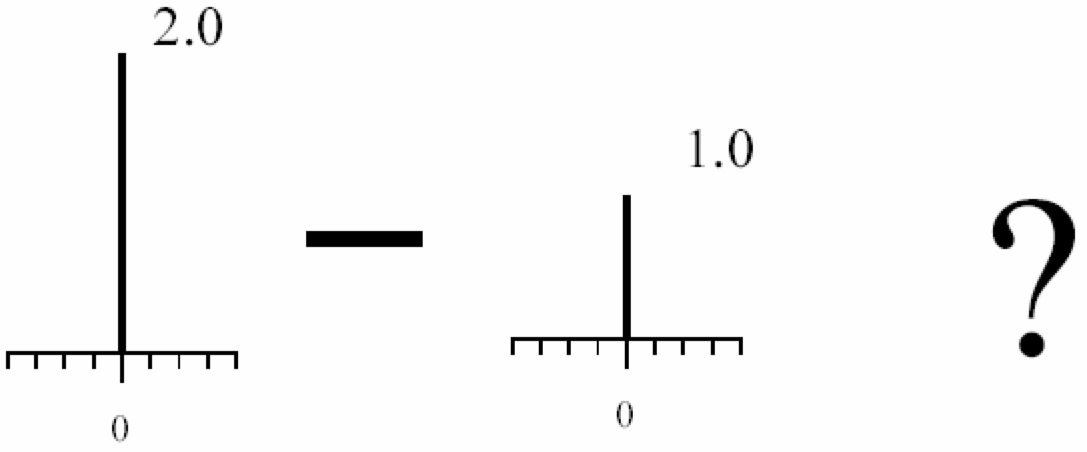
Blur examples



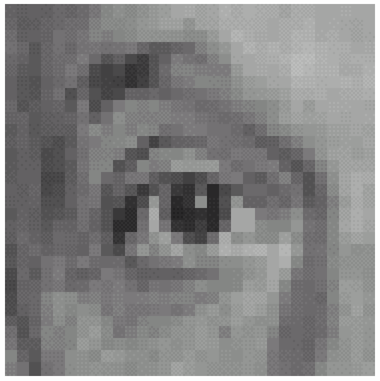
Linear filtering (warm-up slide)

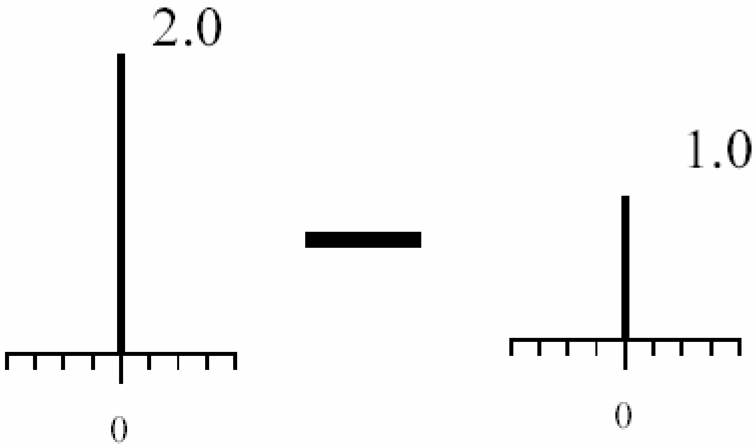


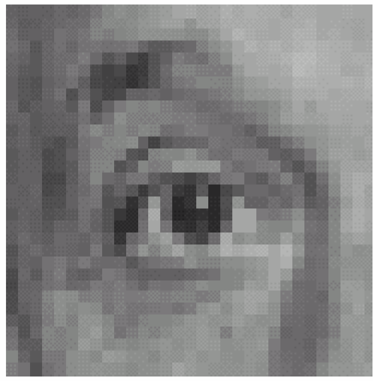
original



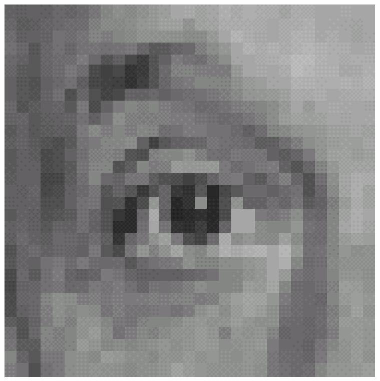
Linear filtering (no change)

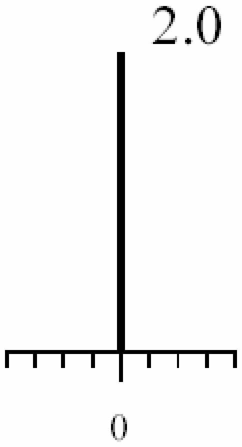

original



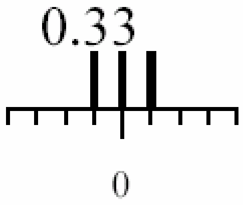

Filtered
(no change)

Linear filtering


original

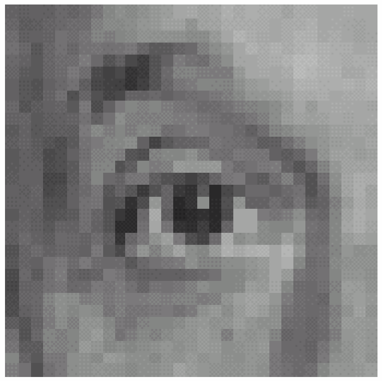


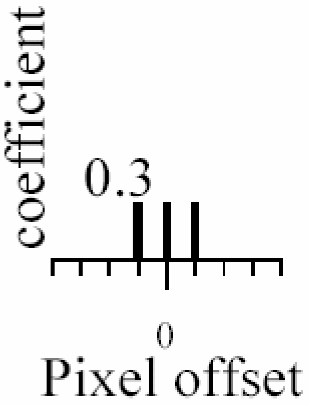
−

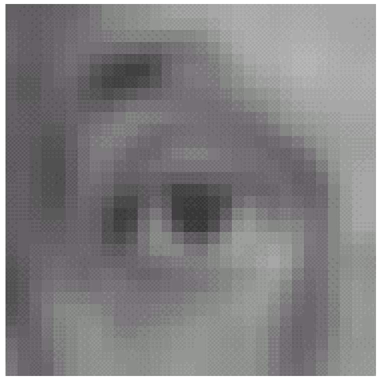


?

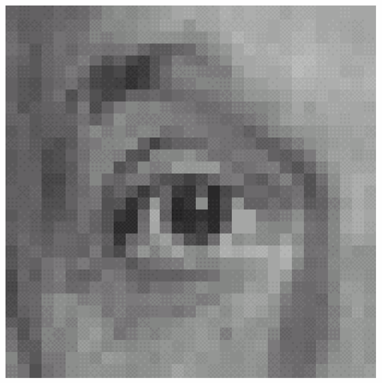
(remember blurring)

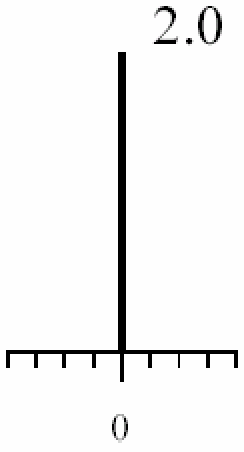

original

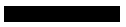


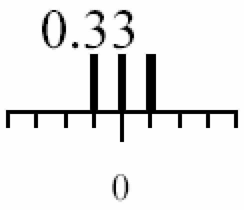

Blurred (filter applied in both dimensions).

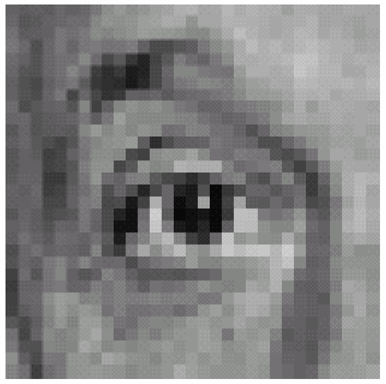
Sharpening


original

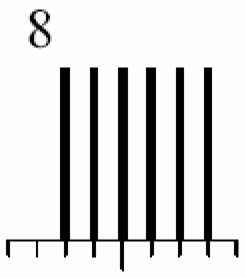




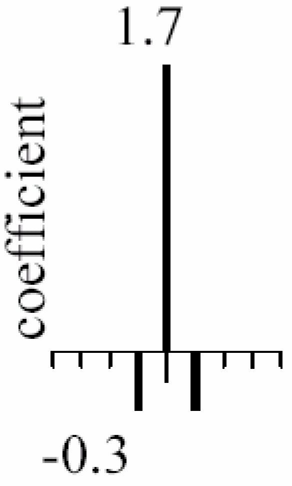


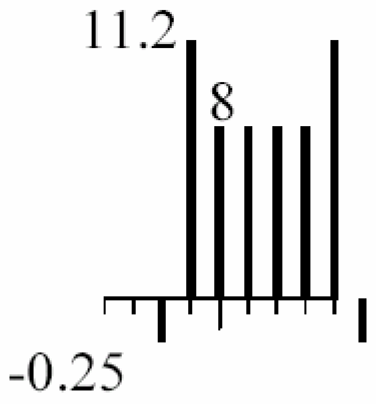

Sharpened original

Sharpening example



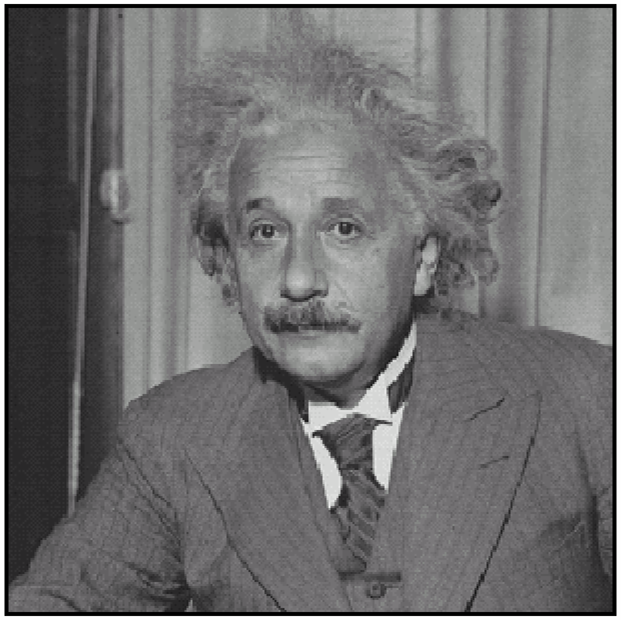
original

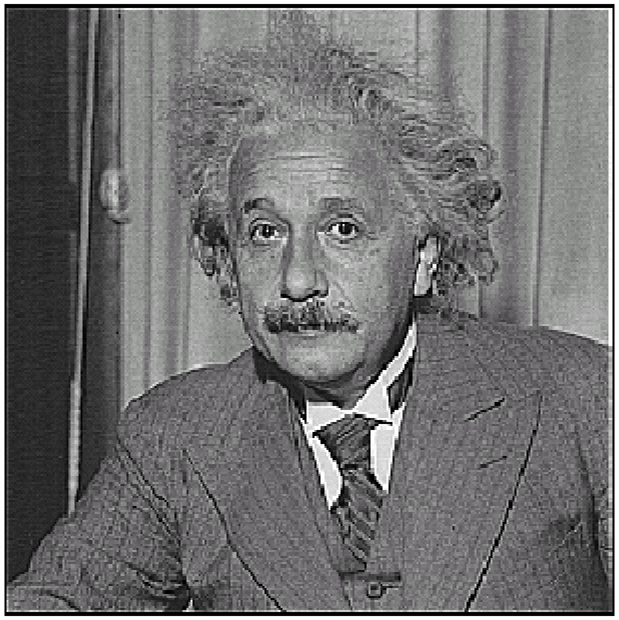




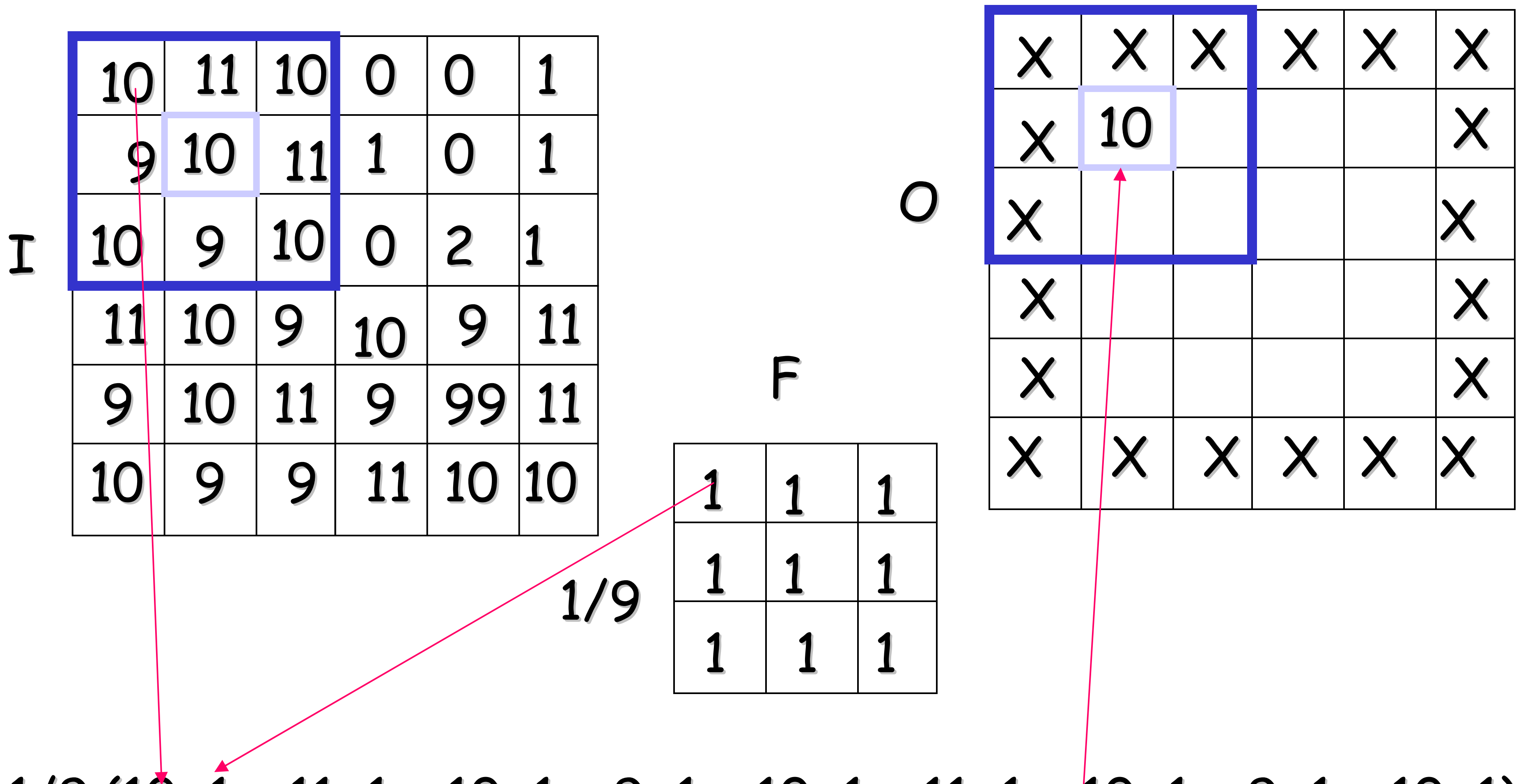
Sharpened (differences are accentuated; constant areas are left untouched).

Sharpening


before


after

Convolution



I

10	11	10	0	0	1
9	10	11	1	0	1
10	9	10	0	2	1
11	10	9	10	9	11
9	10	11	9	99	11
10	9	9	11	10	10

F

1/9

1	1	1
1	1	1
1	1	1

O

X	X	X	X	X	X
X	10				X
X					X
X					X
X					X
X	X	X	X	X	X

1/9.(10x1 + 11x1 + 10x1 + 9x1 + 10x1 + 11x1 + 10x1 + 9x1 + 10x1) =
1/9.(90) = 10

Slide credit: Christopher Rasmussen

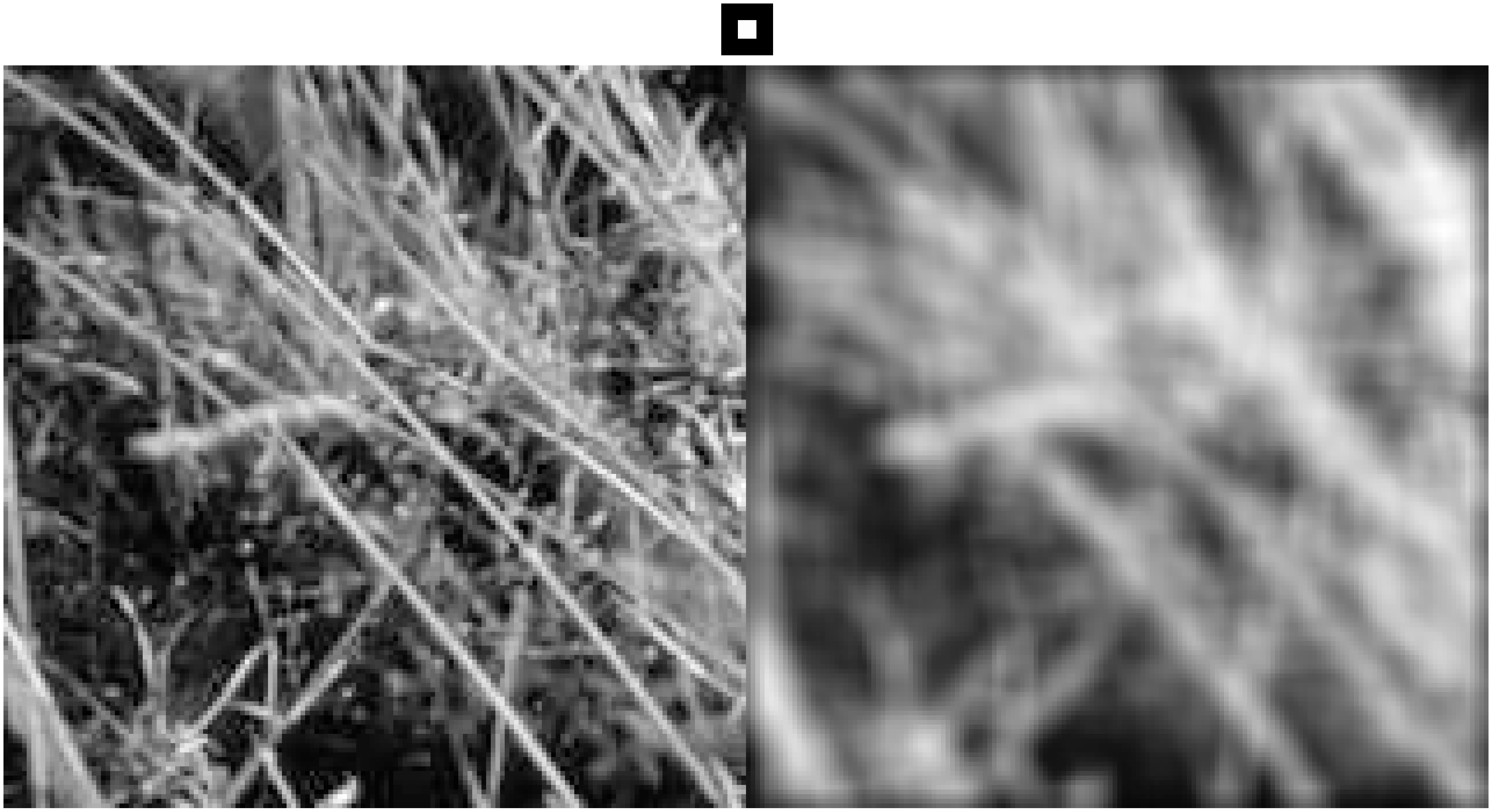
Average filter (box filter)

- Mask with positive entries, that sum to 1.
- Replaces each pixel with an average of its neighborhood.
- If all weights are equal, it is called a **box** filter.

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

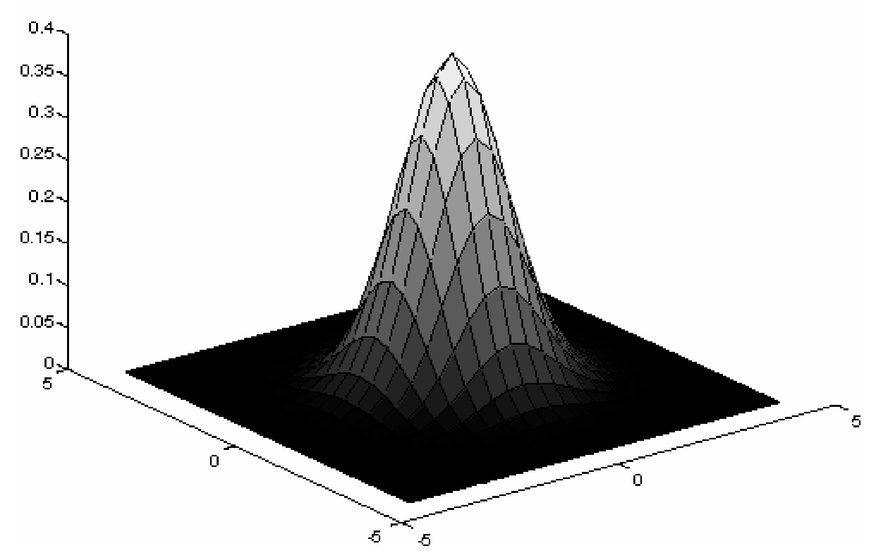
Slide credit: David Lowe (UBC)

Example: Smoothing with a box filter



Smoothing with a Gaussian

- Smoothing with a box actually doesn't compare at all well with a defocussed lens
- Most obvious difference is that a single point of light viewed in a defocussed lens looks like a fuzzy blob; but the averaging process would give a little square.

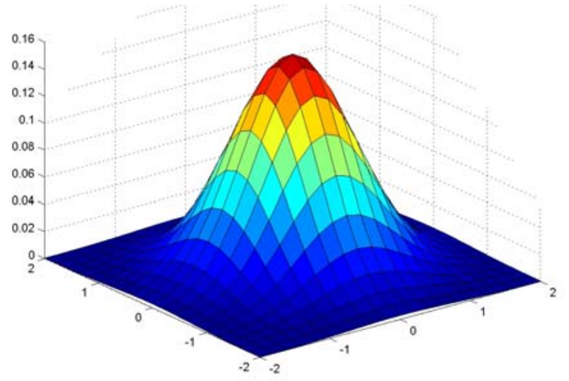


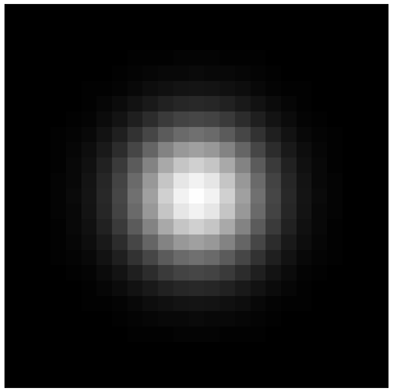
- A Gaussian gives a good model of a fuzzy blob
- It closely models many physical processes (the sum of many small effects)

Slide credit: David Lowe (UBC)

Gaussian Kernel

- Idea: Weight contributions of neighboring pixels by nearness





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

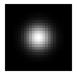
5 x 5, $\sigma = 1$

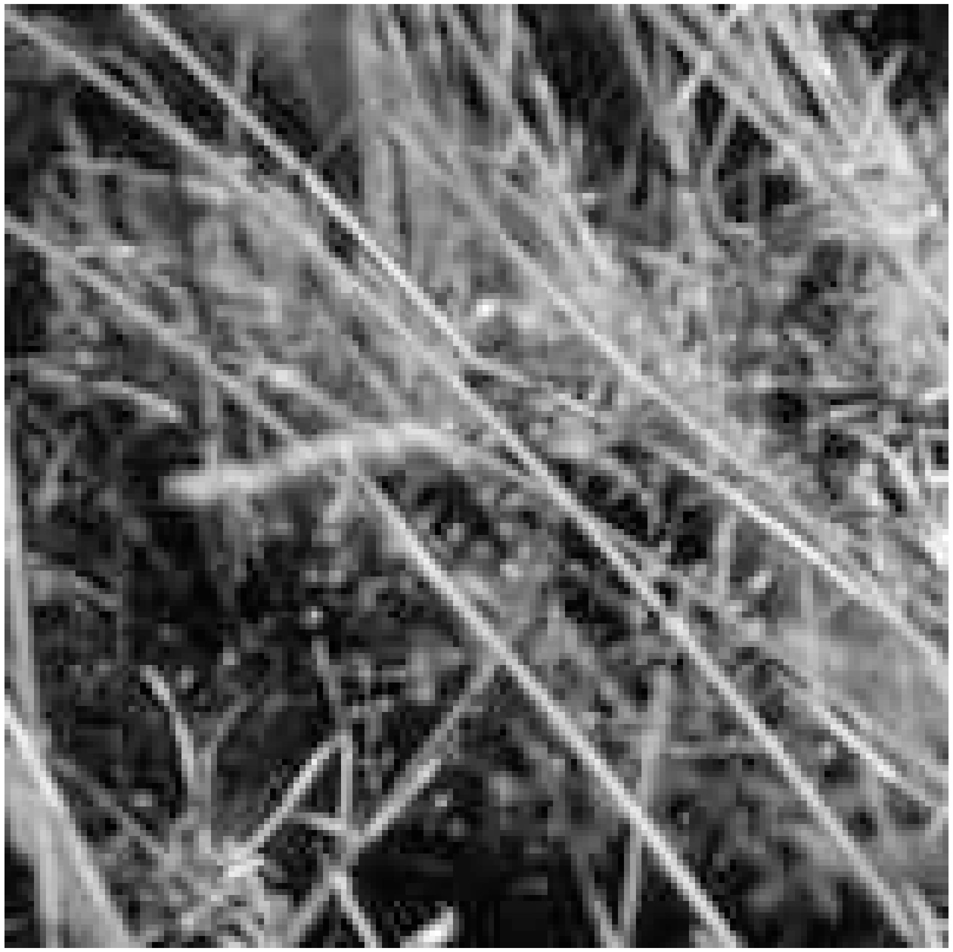
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

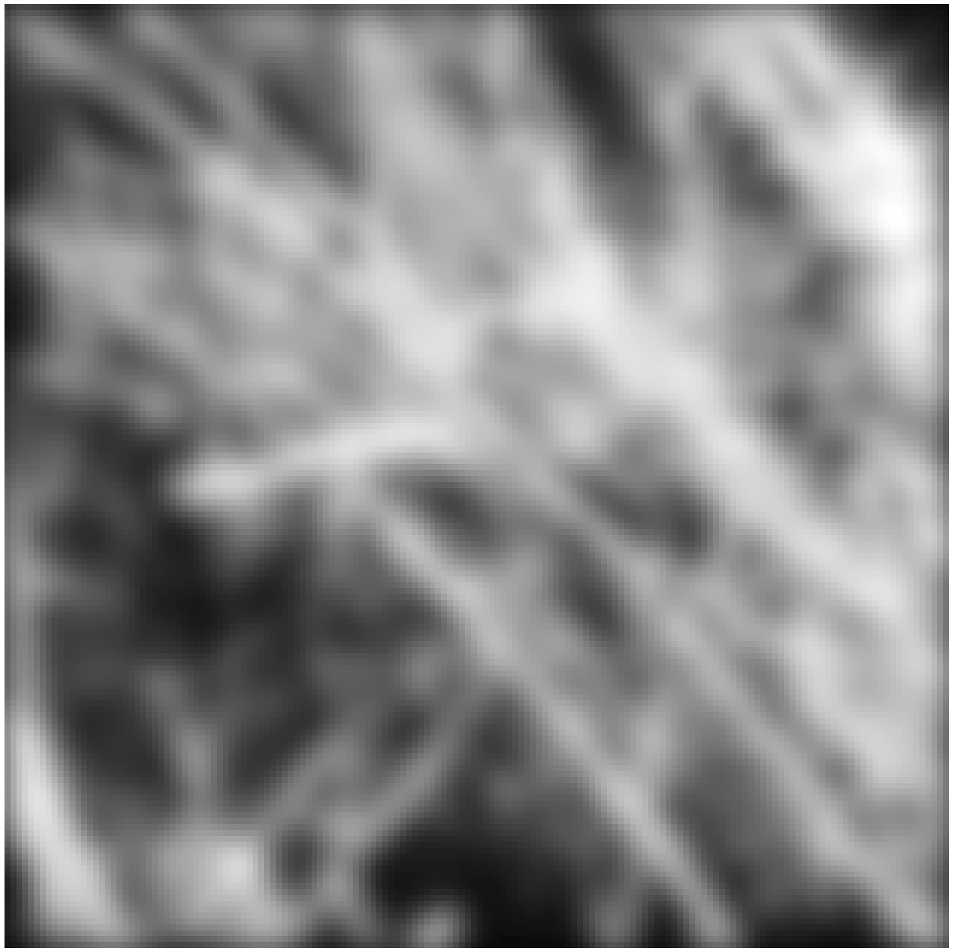
- Constant factor at front makes volume sum to 1 (can be ignored, as we should normalize weights to sum to 1 in any case).

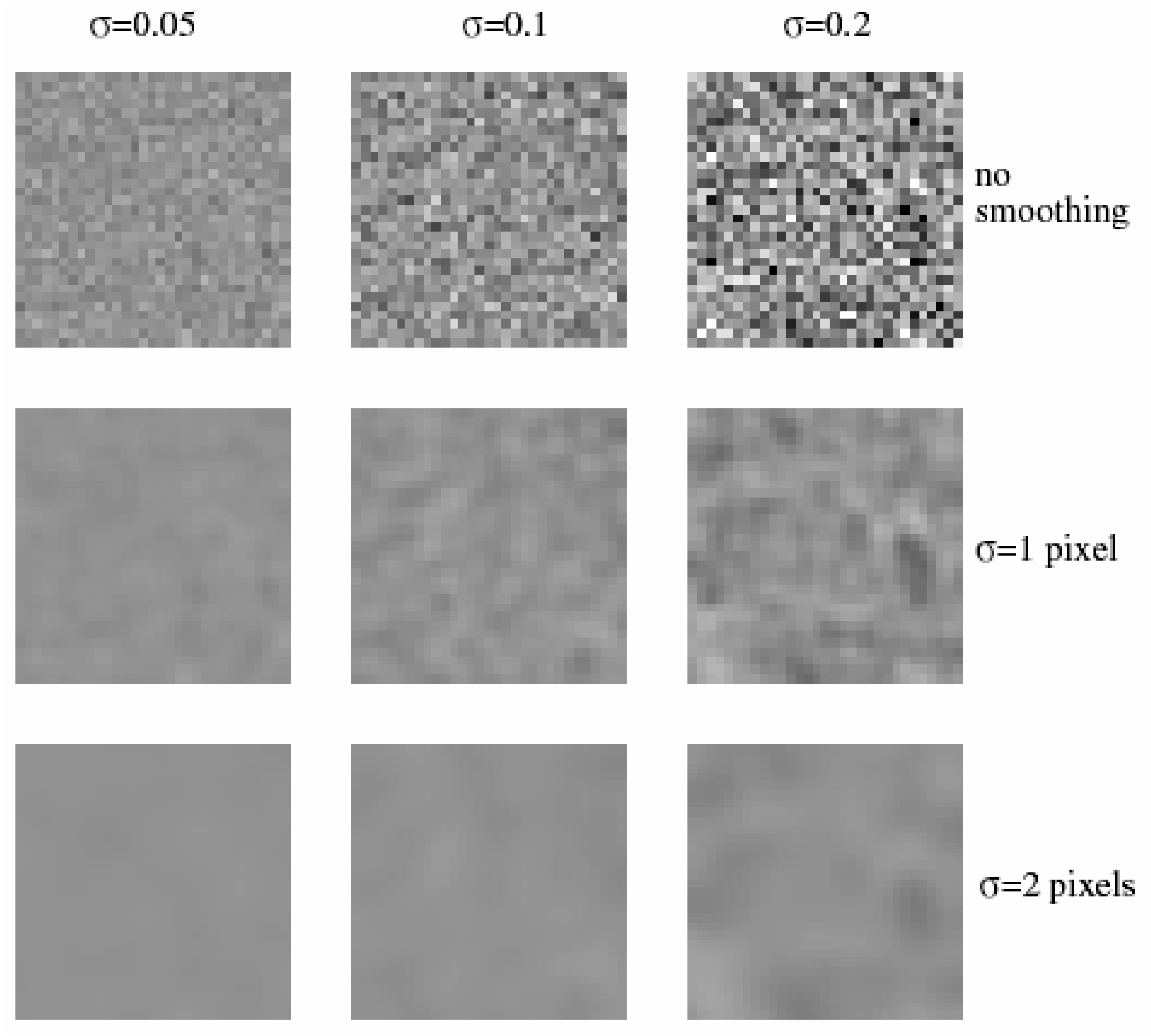
Slide credit: Christopher Rasmussen

Smoothing with a Gaussian









Smoothing reduces pixel noise:

Each row shows smoothing with Gaussians of different width; each column shows different amounts of Gaussian noise.

Efficient Implementation

- Both the BOX filter and the Gaussian filter are **separable** into two 1D convolutions:
  - First convolve each row with a 1D filter
  - Then convolve each column with a 1D filter.

Differentiation and convolution

- Recall, for 2D function, f(x,y):

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon, y)}{\varepsilon} - \frac{f(x,y)}{\varepsilon} \right)$$

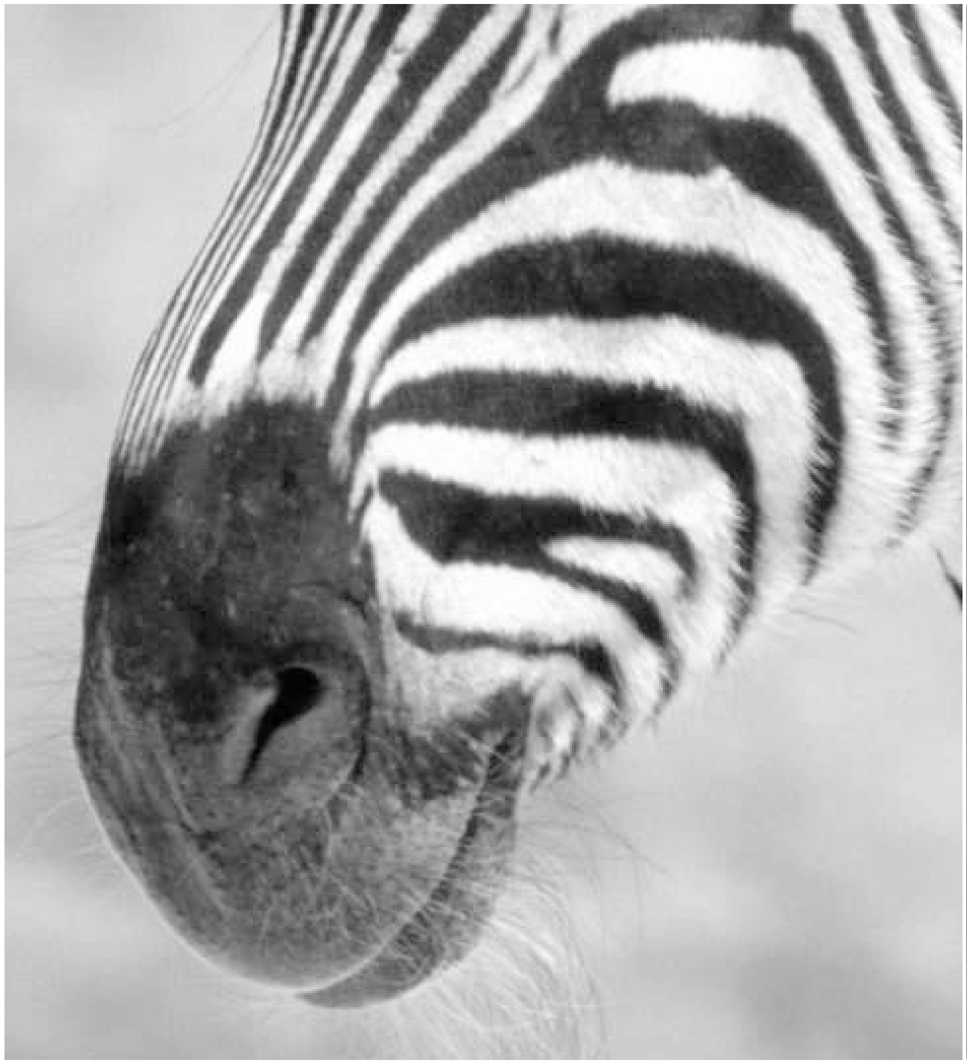
- This is linear and shift invariant, so must be the result of a convolution.

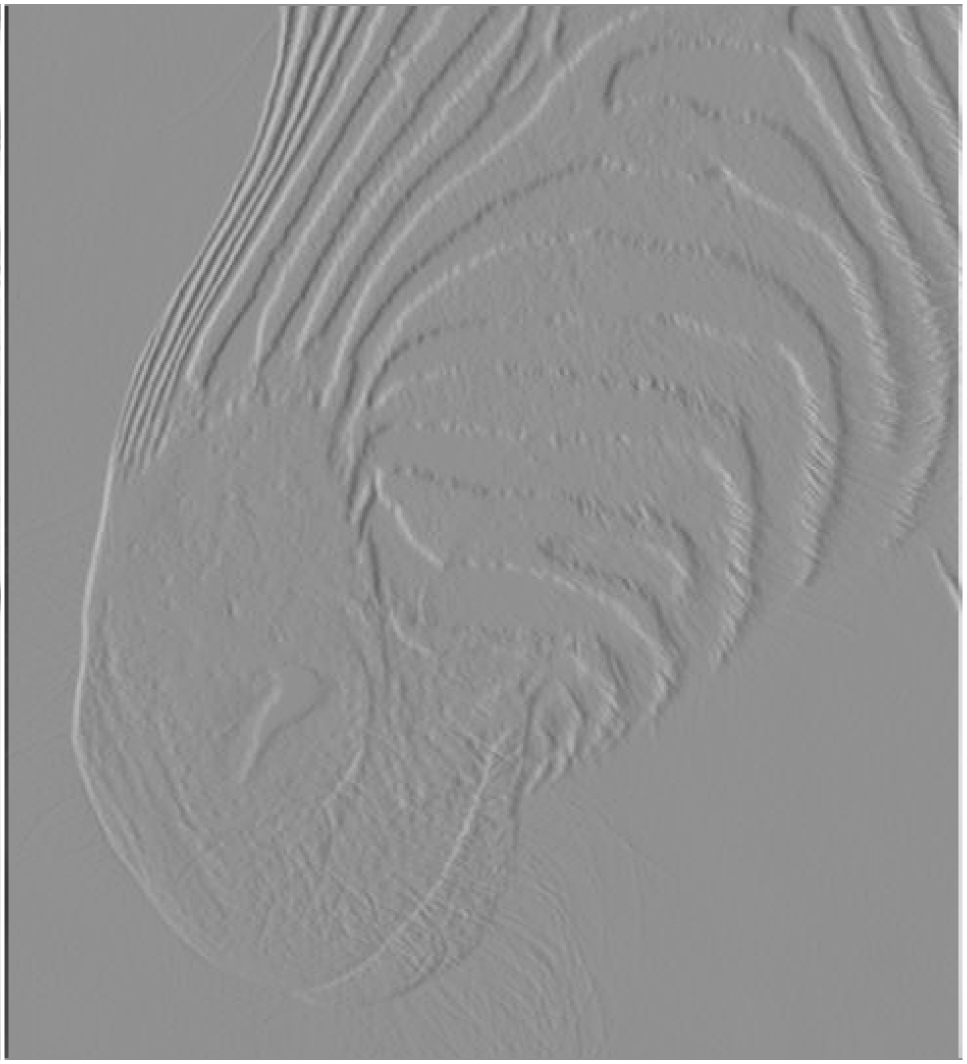
- We could approximate this as

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}, y) - f(x_n, y)}{\Delta x}$$

(which is obviously a convolution)

Vertical gradients from finite differences





Shift invariant linear systems

- 3 properties
  - Superposition
  - Scaling
  - Shift invariance

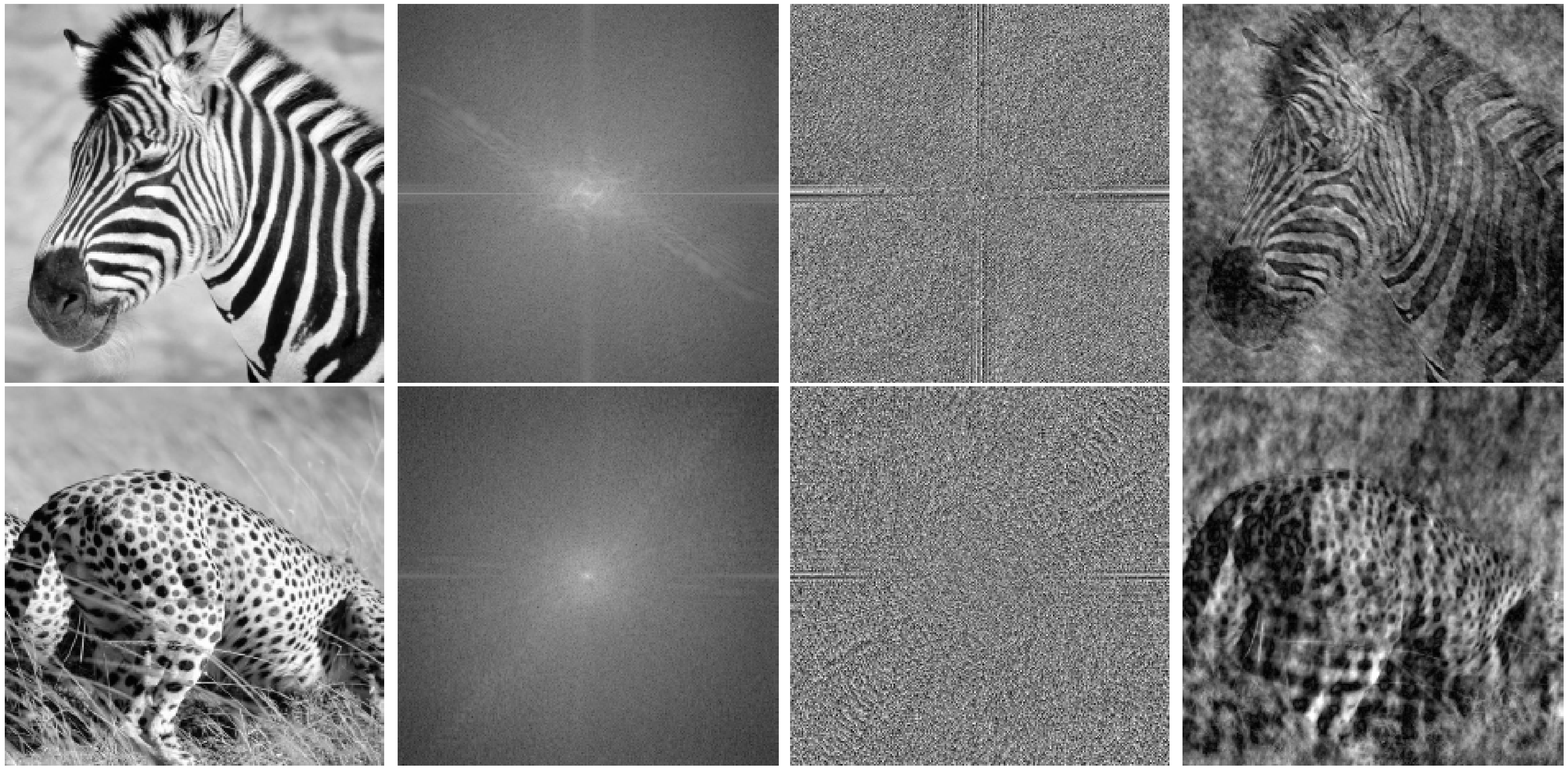
Discrete convolution

- 1D
- 2D
- “edge effects” in discrete convolution

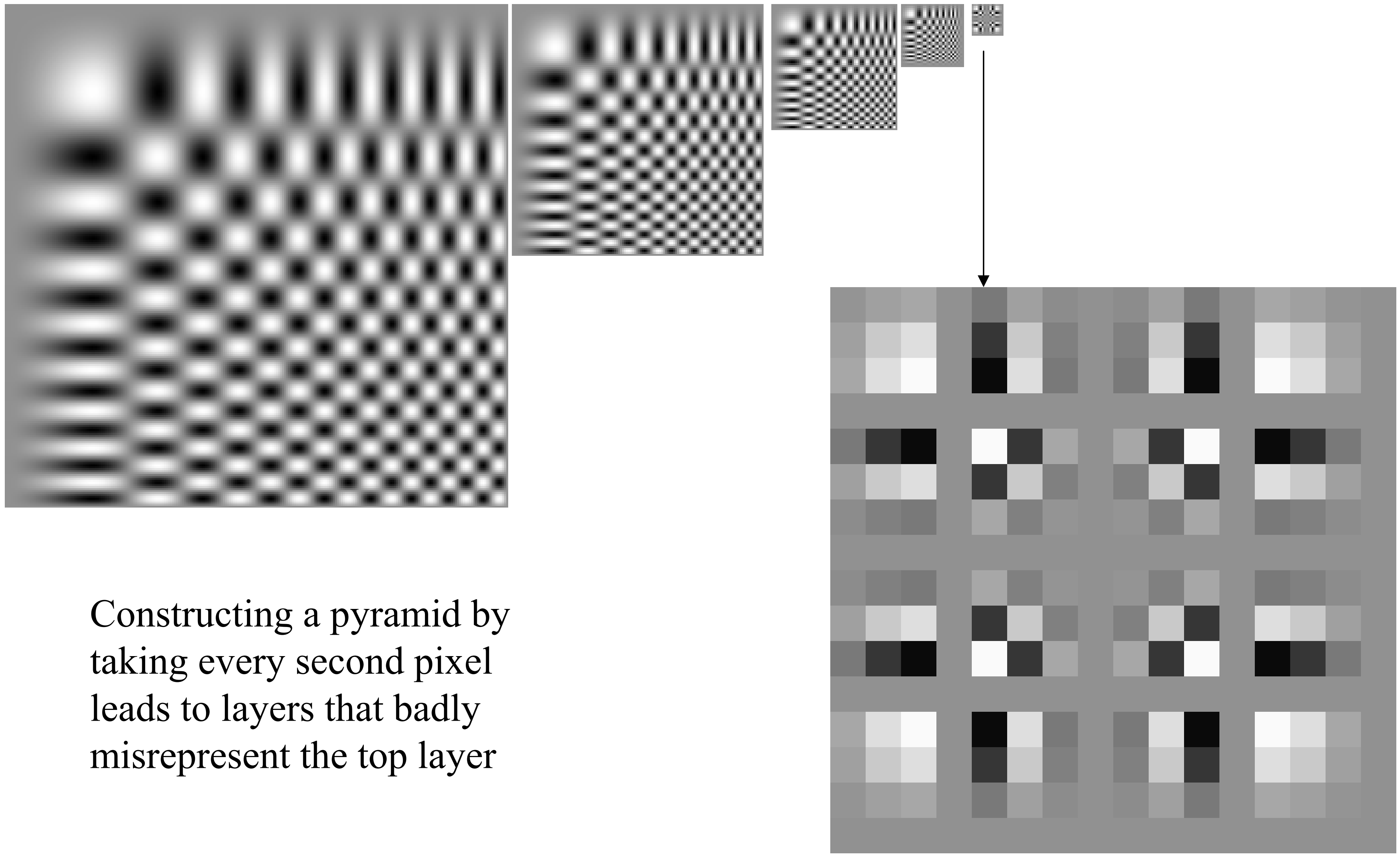
Spatial frequency and Fourier Transform

$$\mathcal{F}(g(x,y))(u,v) = \iint_{-\infty}^{\infty} g(x,y)e^{-i2\pi(ux+vy)}\,dx\,dy$$

Function	Fourier transform
$g(x,y)$	$\int\int_{-\infty}^{\infty} g(x,y)e^{-i2\pi(ux+vy)}dxdy$
$\int\int_{-\infty}^{\infty} \mathcal{F}(g(x,y))(u,v)e^{i2\pi(ux+vy)}dudv$	$\mathcal{F}(g(x,y))(u,v)$
$\delta(x,y)$	1
$\frac{\partial f}{\partial x}(x,y)$	$u\mathcal{F}(f)(u,v)$
$0.5\delta(x+a,y)+0.5\delta(x-a,y)$	$\cos 2\pi au$
$e^{-\pi(x^2+y^2)}$	$e^{-\pi(u^2+v^2)}$
$box_1(x,y)$	$\frac{\sin u}{u}\frac{\sin v}{v}$
$f(ax,by)$	$\frac{\mathcal{F}(f)(u/a,v/b)}{ab}$
$\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\delta(x-i,y-j)$	$\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\delta(u-i,v-j)$
$(f**g)(x,y)$	$\mathcal{F}(f)\mathcal{F}(g)(u,v)$
$f(x-a,y-b)$	$e^{-i2\pi(au+bv)}\mathcal{F}(f)$
$f(x\cos\theta-y\sin\theta,x\sin\theta+y\cos\theta)$	$\mathcal{F}(f)(u\cos\theta-v\sin\theta,u\sin\theta+v\cos\theta)$



Sampling and aliasing



Constructing a pyramid by taking every second pixel leads to layers that badly misrepresent the top layer

Sampling in 1D

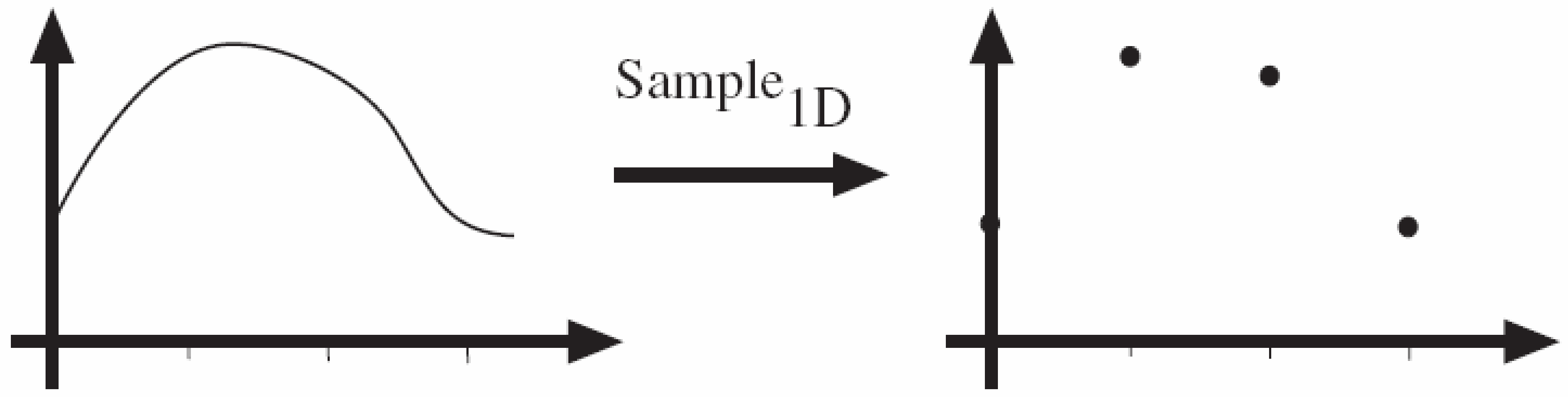
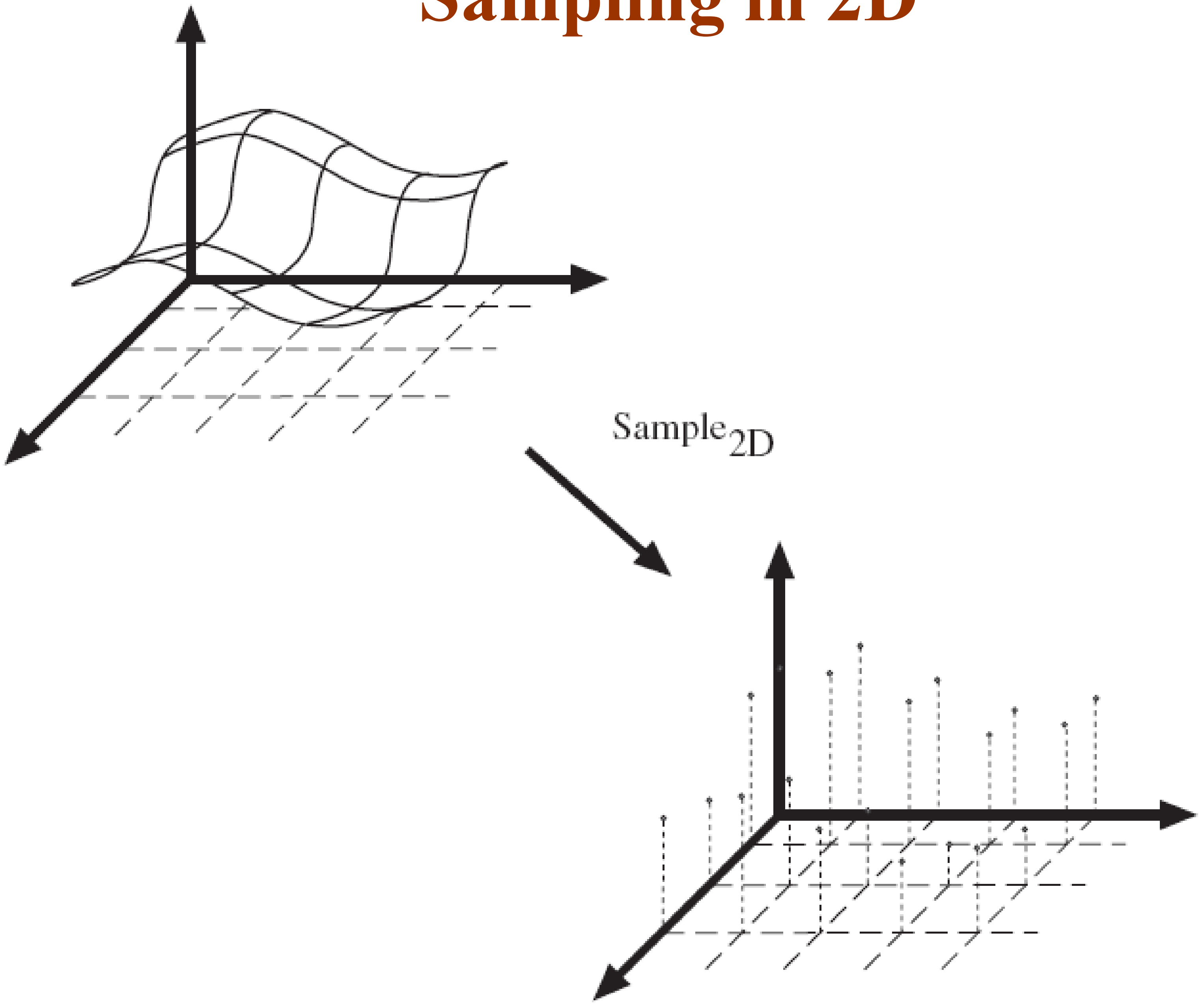
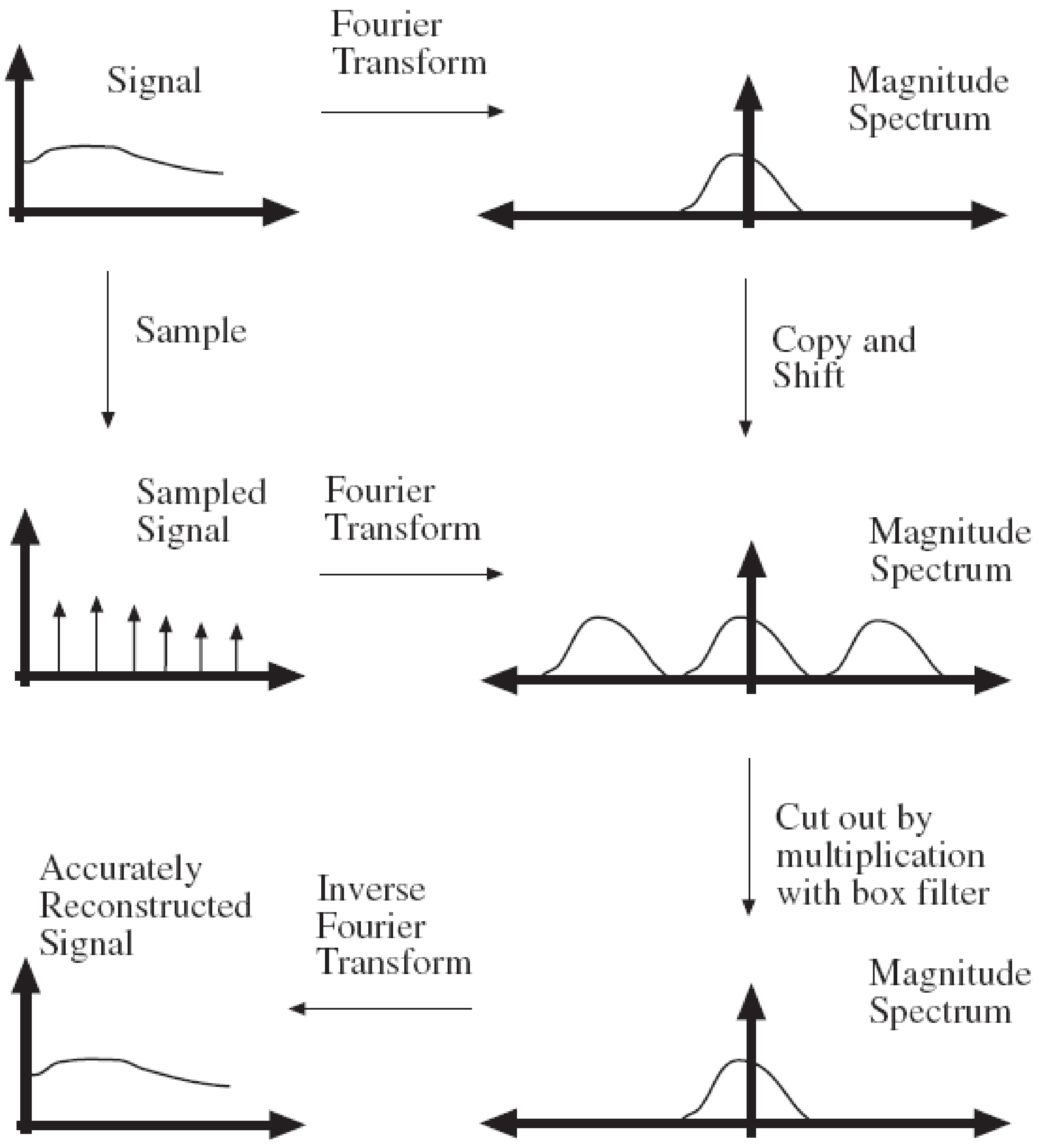
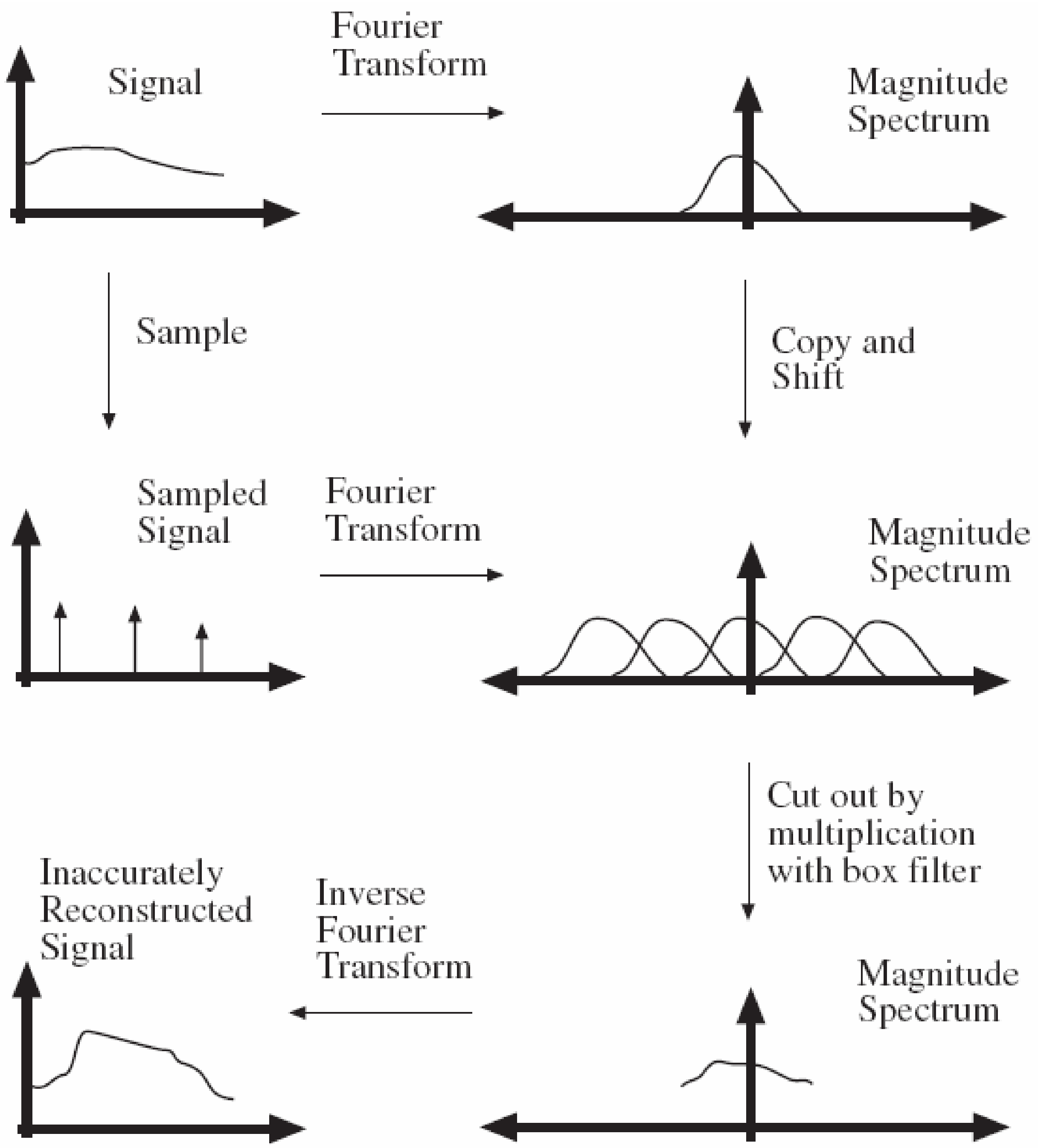


FIGURE 7.8: Sampling in 1D takes a function, and returns a vector whose elements are values of that function at the sample points, as the top figures show. For our purposes, it is enough that the sample points be integer values of the argument. We allow the vector to be infinite dimensional, and have negative as well as positive indices.

Sampling in 2D

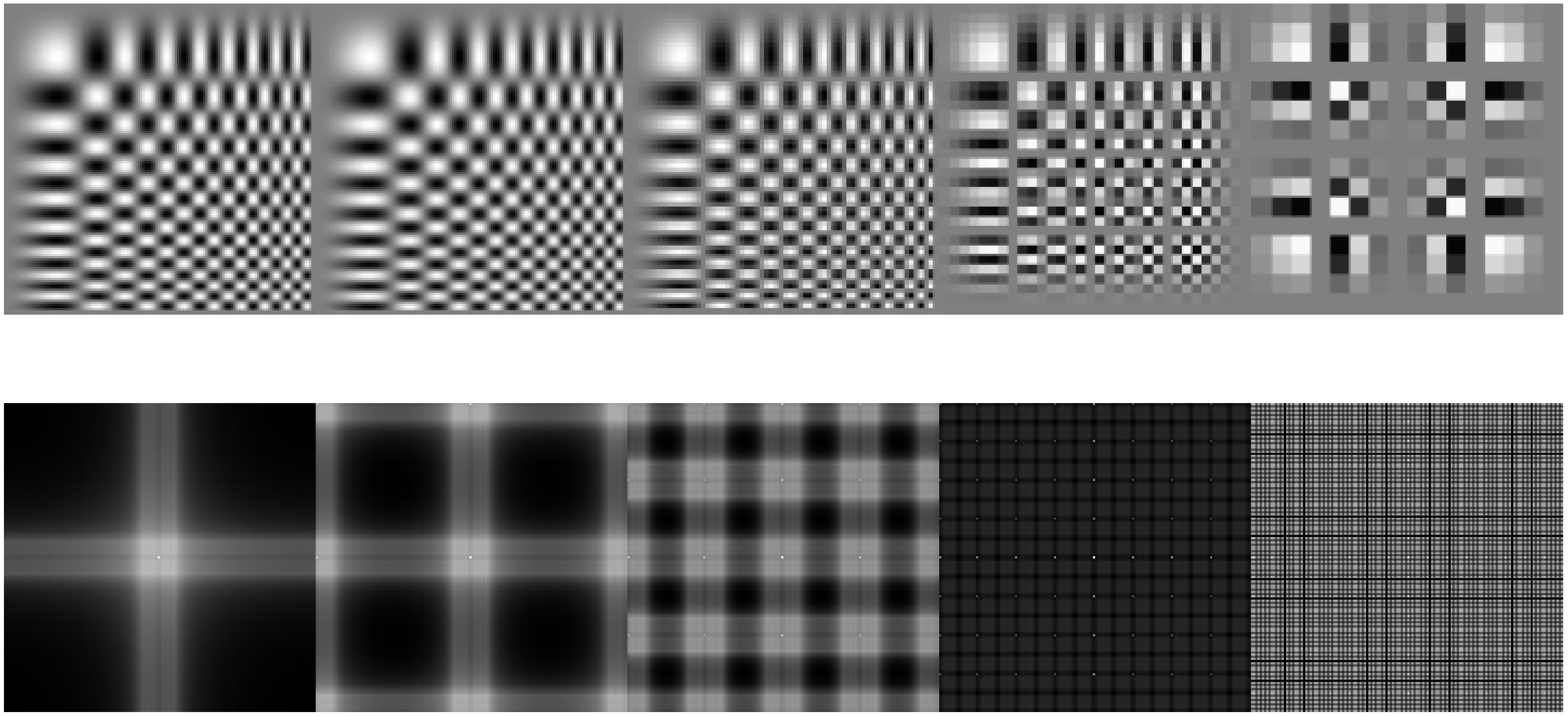



Signal
Fourier Transform
Magnitude Spectrum
Sample
Copy and Shift
Sampled Signal
Fourier Transform
Magnitude Spectrum
Cut out by multiplication with box filter
Accurately Reconstructed Signal
Inverse Fourier Transform
Magnitude Spectrum


Signal
Fourier Transform
Magnitude Spectrum
Sample
Copy and Shift
Sampled Signal
Fourier Transform
Magnitude Spectrum
Cut out by multiplication with box filter
Inaccurately Reconstructed Signal
Inverse Fourier Transform
Magnitude Spectrum

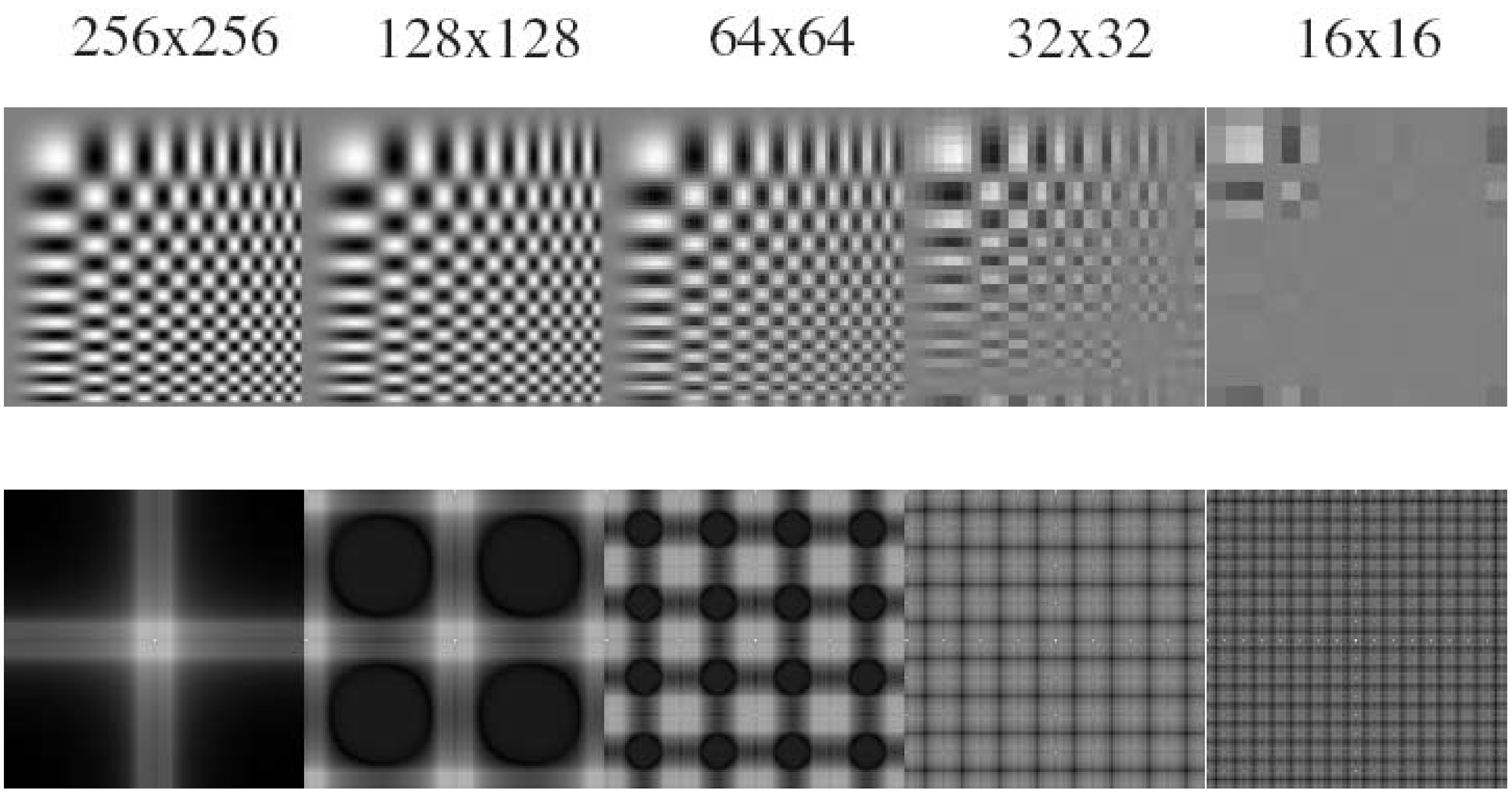
Aliasing!

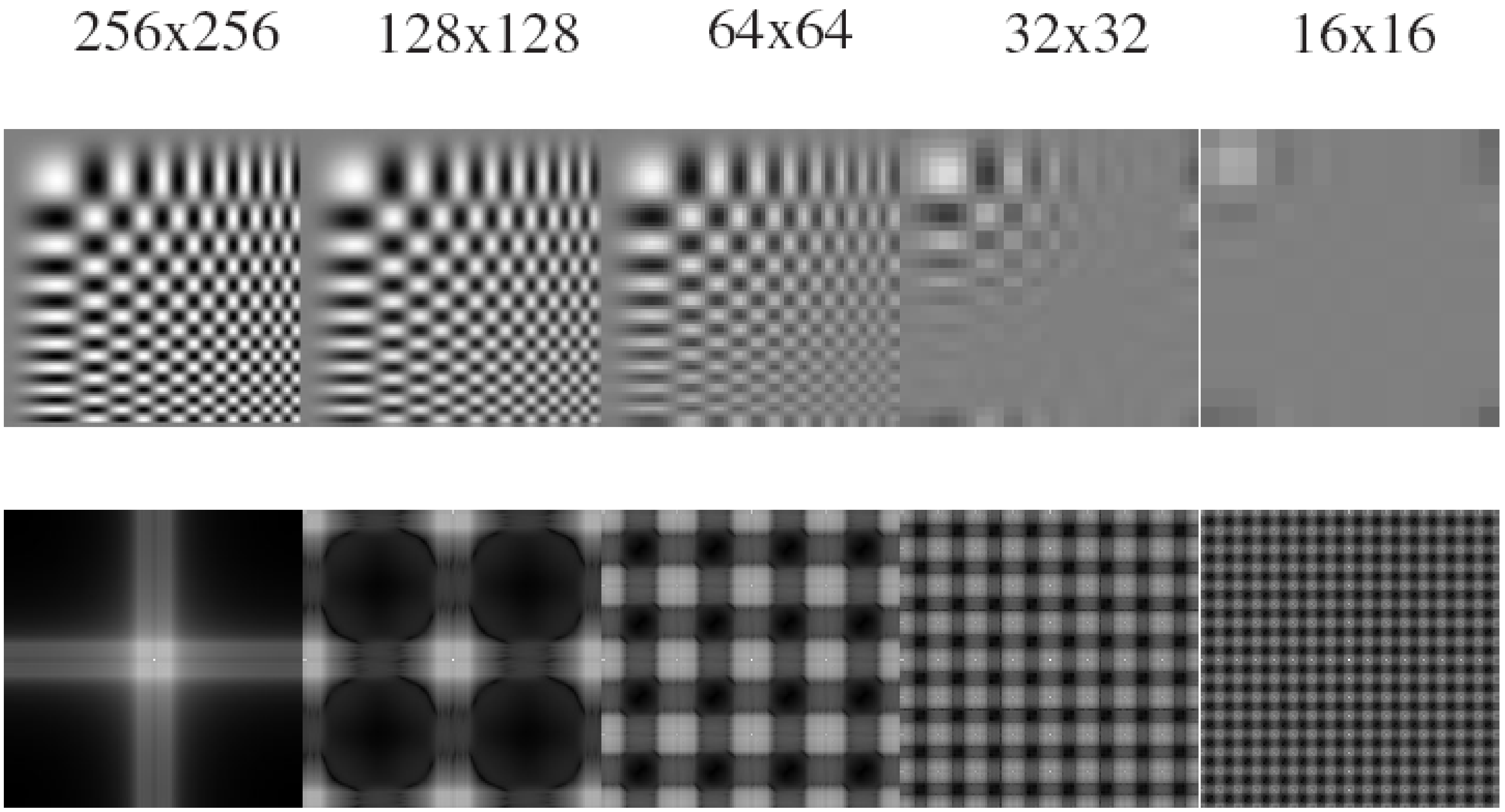
256x256 128x128 64x64 32x32 16x16



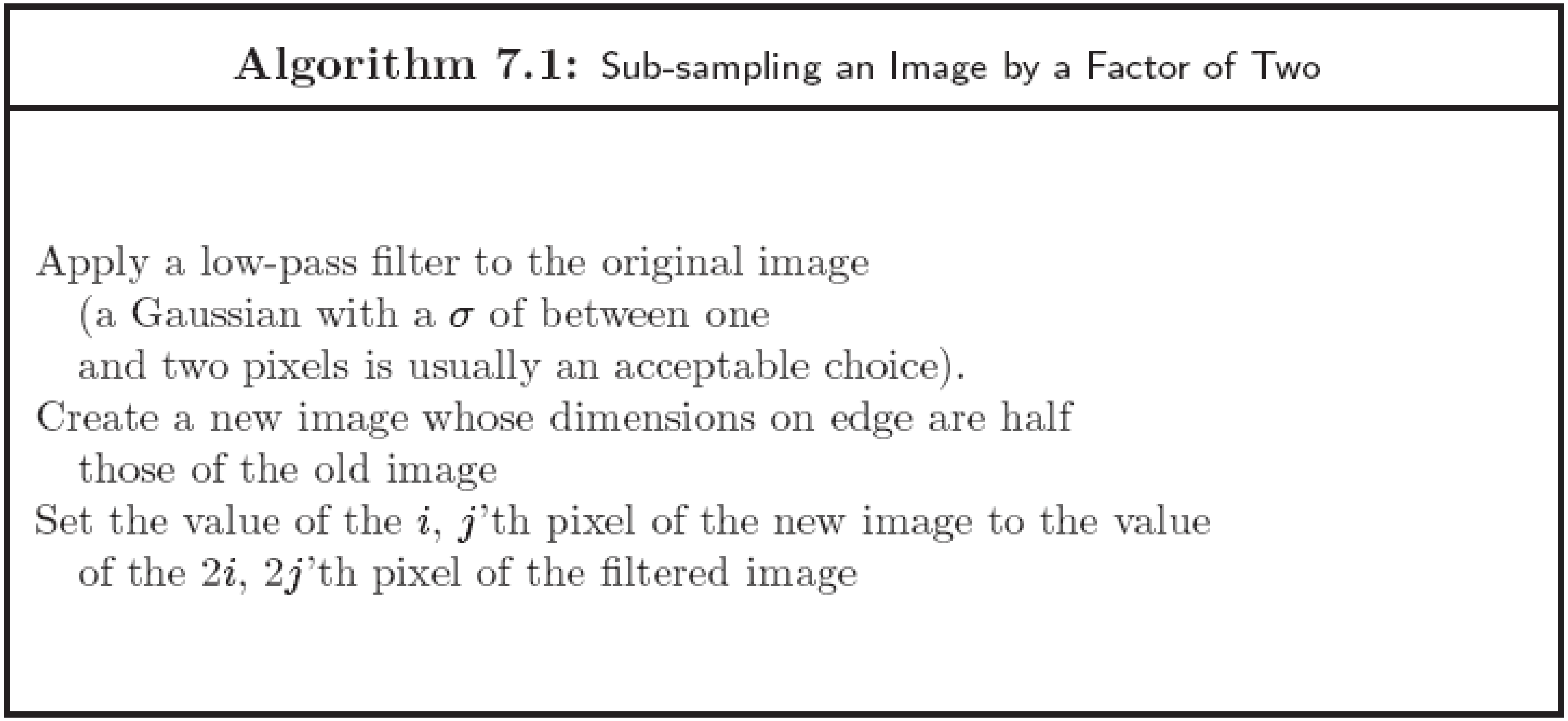
Smoothing and resampling

- Nyquist's theorem




256x256
128x128
64x64
32x32
16x16

Algorithm



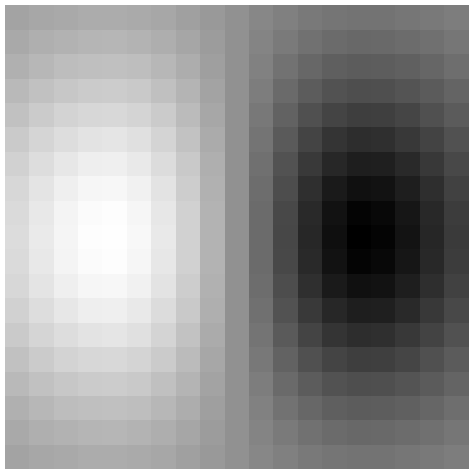
Algorithm 7.1: Sub-sampling an Image by a Factor of Two

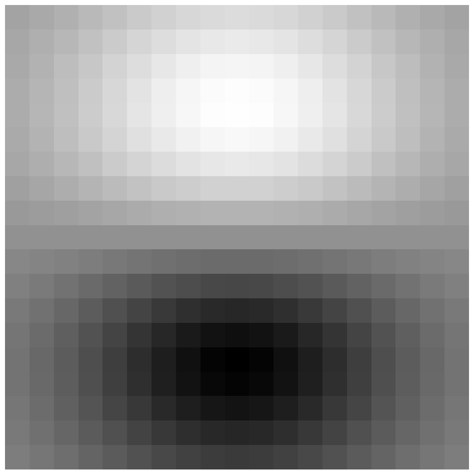
```
Apply a low-pass filter to the original image
   (a Gaussian with a $\sigma$ of between one
   and two pixels is usually an acceptable choice).
Create a new image whose dimensions on edge are half
   those of the old image
Set the value of the $i$, $j$'th pixel of the new image to the value
   of the $2i$, $2j$'th pixel of the filtered image
```

Filters are templates

- Applying a filter at some point can be seen as taking a dot-product between the image and some vector
- Filtering the image is a set of dot products

- Insight
  - filters look like the effects they are intended to find
  - filters find effects they look like





Slide credit: David Lowe (UBC)

Normalized correlation

- Think of filters as a dot product of the filter vector with the image region
  - Now measure the angle between the vectors

$$a \cdot b = |a||b| \cos\theta$$

  - Angle (similarity) between vectors can be measured by normalizing length of each vector to 1.
  - Normalized correlation: divide each correlation by square root of sum of squared values (length)

Slide credit: David Lowe (UBC)

We need scaled representations

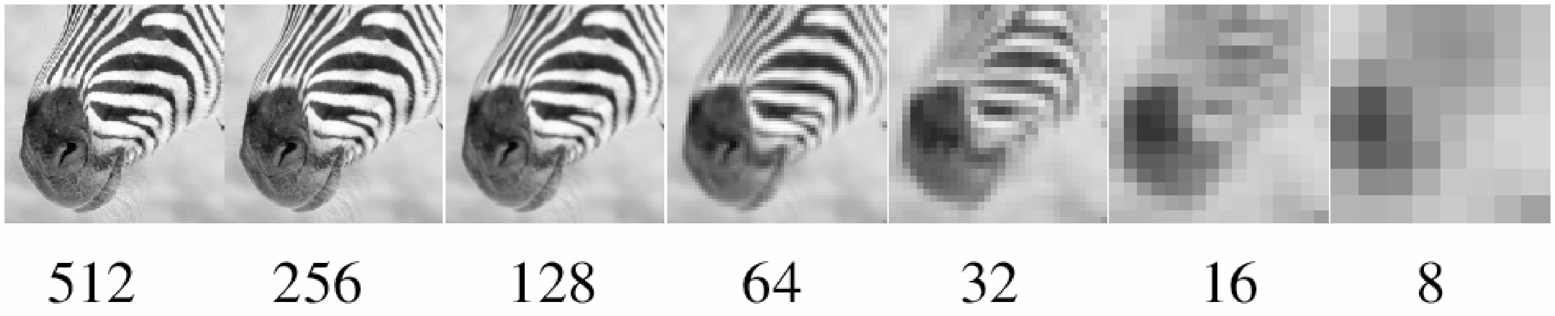
- Find template matches at all scales
  - e.g., when finding hands or faces, we don’t know what size they will be in a particular image
  - Template size is constant, but image size changes
- Efficient search for correspondence
  - look at coarse scales, then refine with finer scales
  - much less cost, but may miss best match
- Examining all levels of detail
  - Find edges with different amounts of blur
  - Find textures with different spatial frequencies (levels of detail)

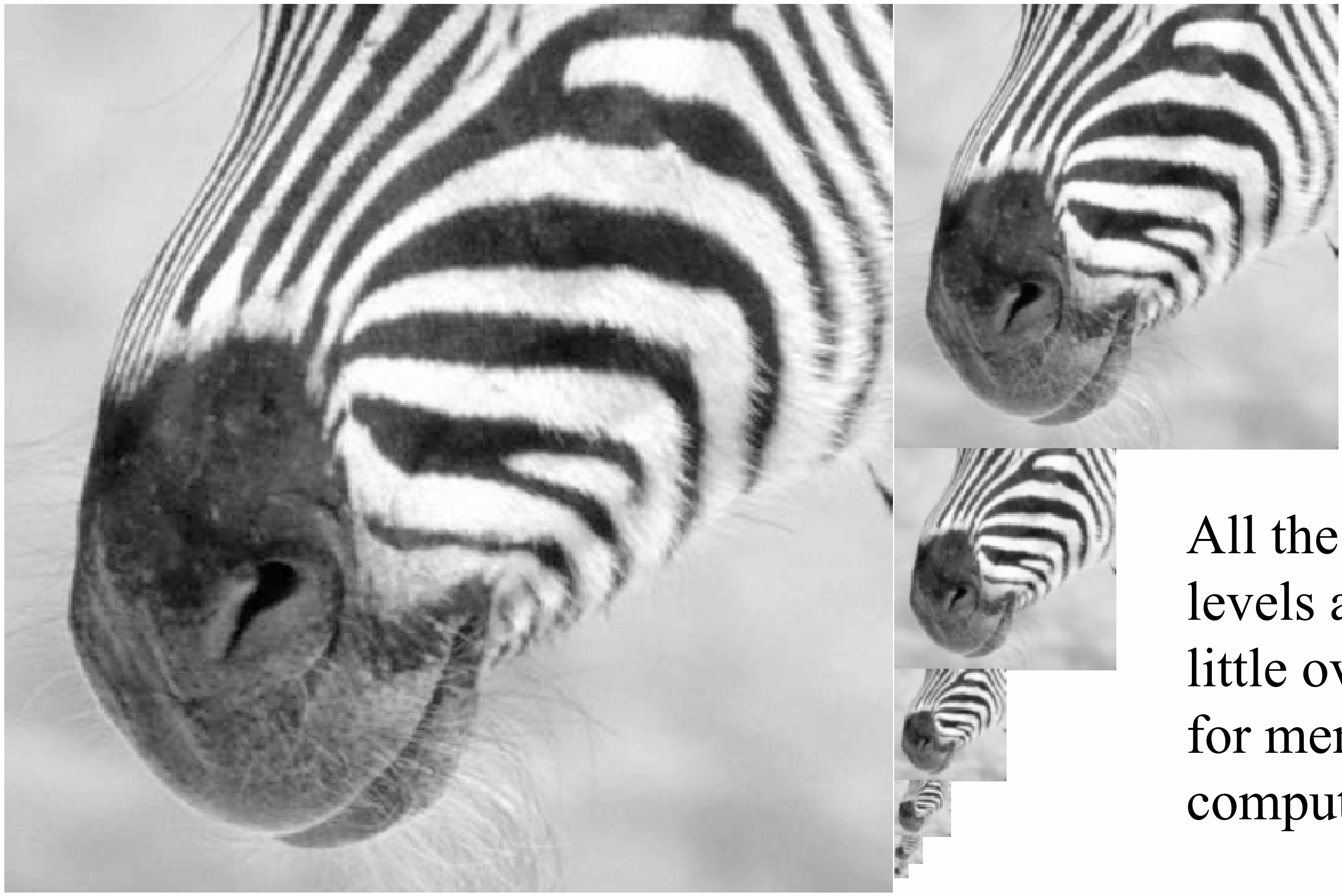
Slide credit: David Lowe (UBC)

The Gaussian pyramid

- Create each level from previous one:
  - smooth and sample
- Smooth with Gaussians, in part because
  - a Gaussian*Gaussian = another Gaussian
  - G(x) * G(y) = G(sqrt($x^2 + y^2$))

Slide credit: David Lowe (UBC)





All the extra levels add very little overhead for memory or computation!

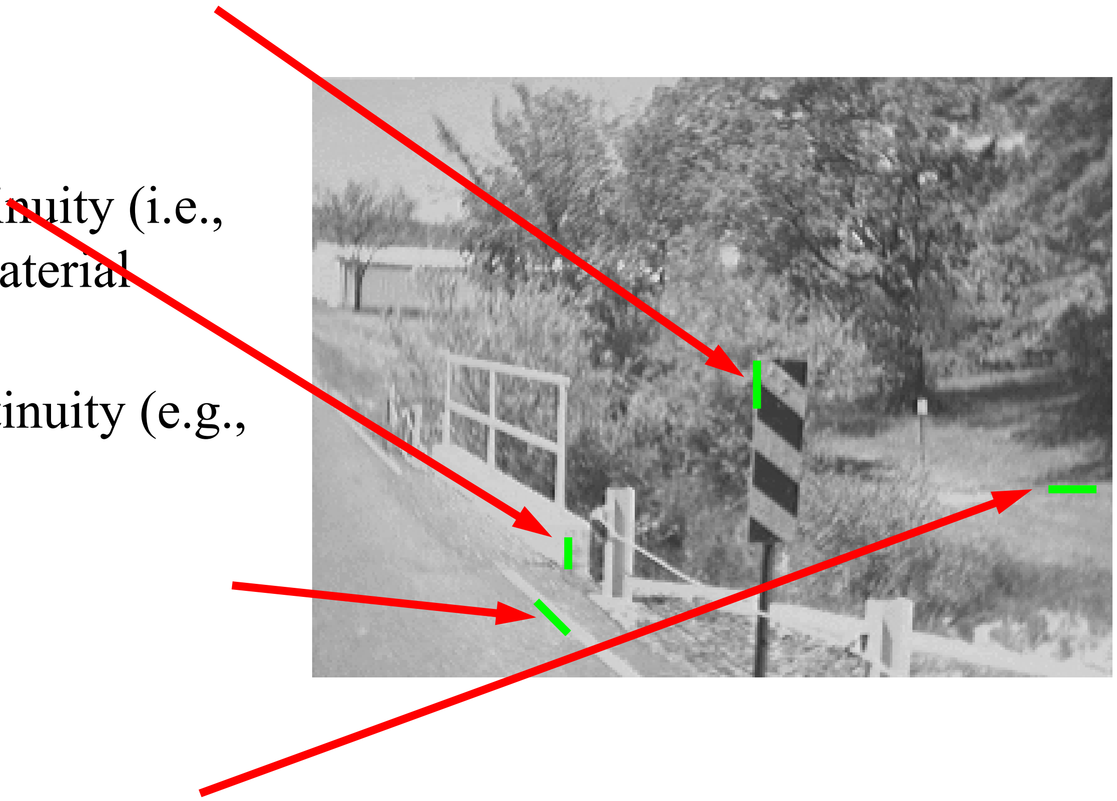
Edge and Corner Detection

- **Goal:** Identify sudden changes (discontinuities) in an image
- This is where most shape information is encoded
- **Example:** artist's line drawing (but artist is also using object-level knowledge)

Slide credit: David Lowe (UBC)

What causes an edge?

- Depth discontinuity
- Surface orientation discontinuity
- Reflectance discontinuity (i.e., change in surface material properties)
- Illumination discontinuity (e.g., shadow)

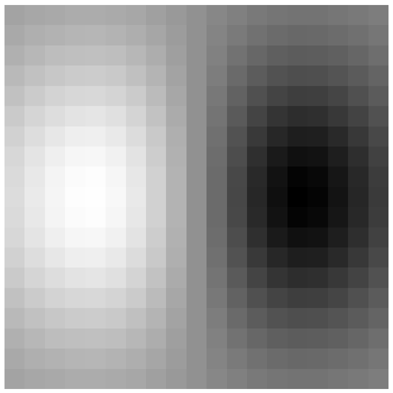
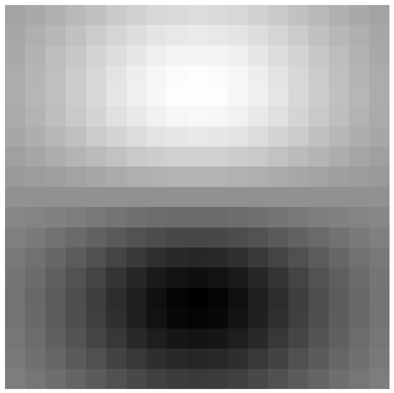


Slide credit: Christopher Rasmussen

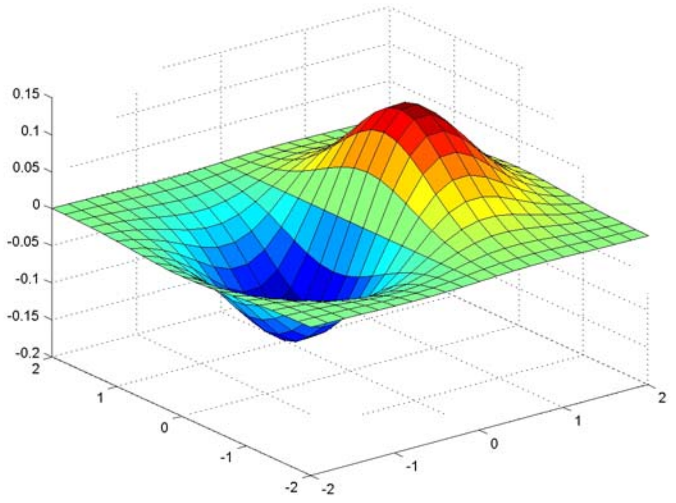
Smoothing and Differentiation

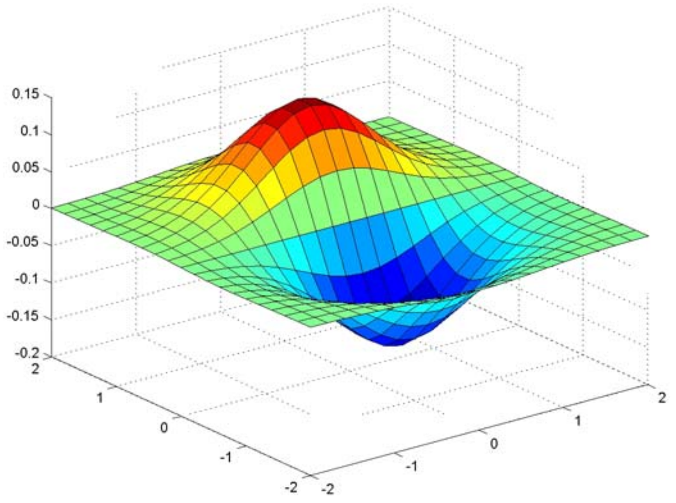
- **Edge:** a location with high gradient (derivative)
- Need smoothing to reduce noise prior to taking derivative
- Need two derivatives, in x and y direction.
- We can use derivative of Gaussian filters
    - because differentiation is convolution, and convolution is associative:

$$D * (G * I) = (D * G) * I$$

Derivative of Gaussian

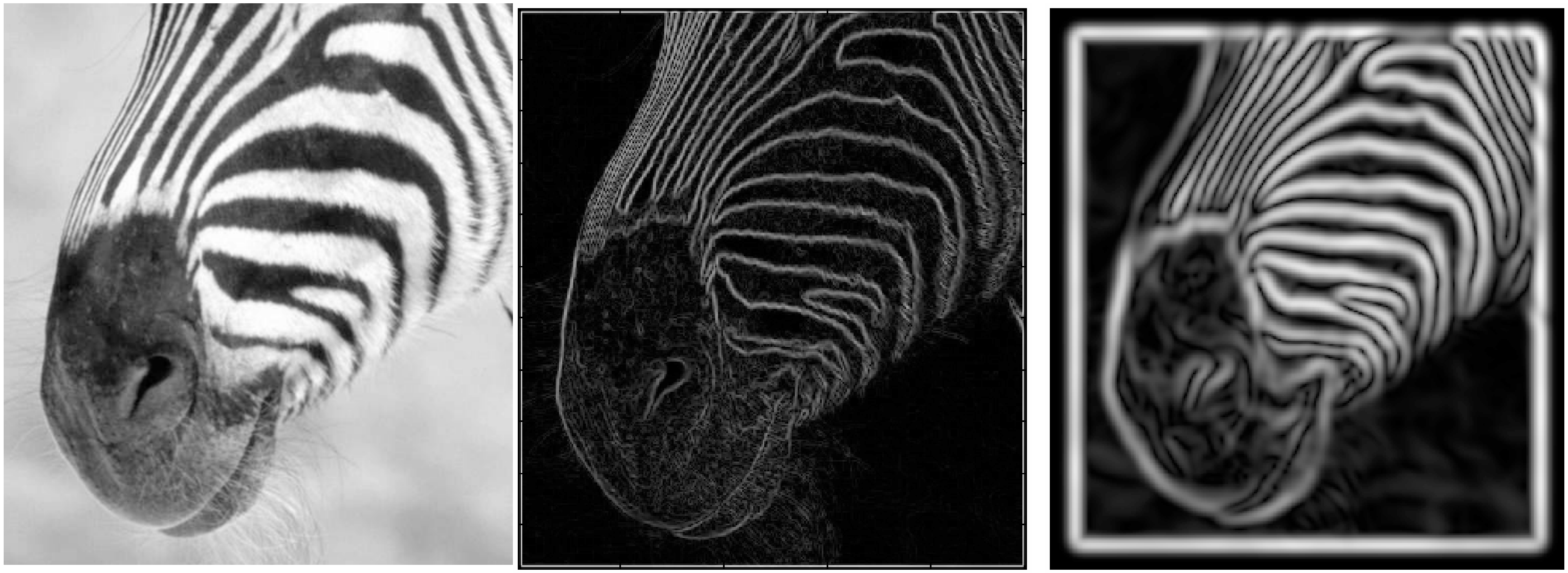




$$\frac{\partial}{\partial x} G_\sigma$$

$$\frac{\partial}{\partial y} G_\sigma$$

Slide credit: Christopher Rasmussen



Scale

Increased smoothing:

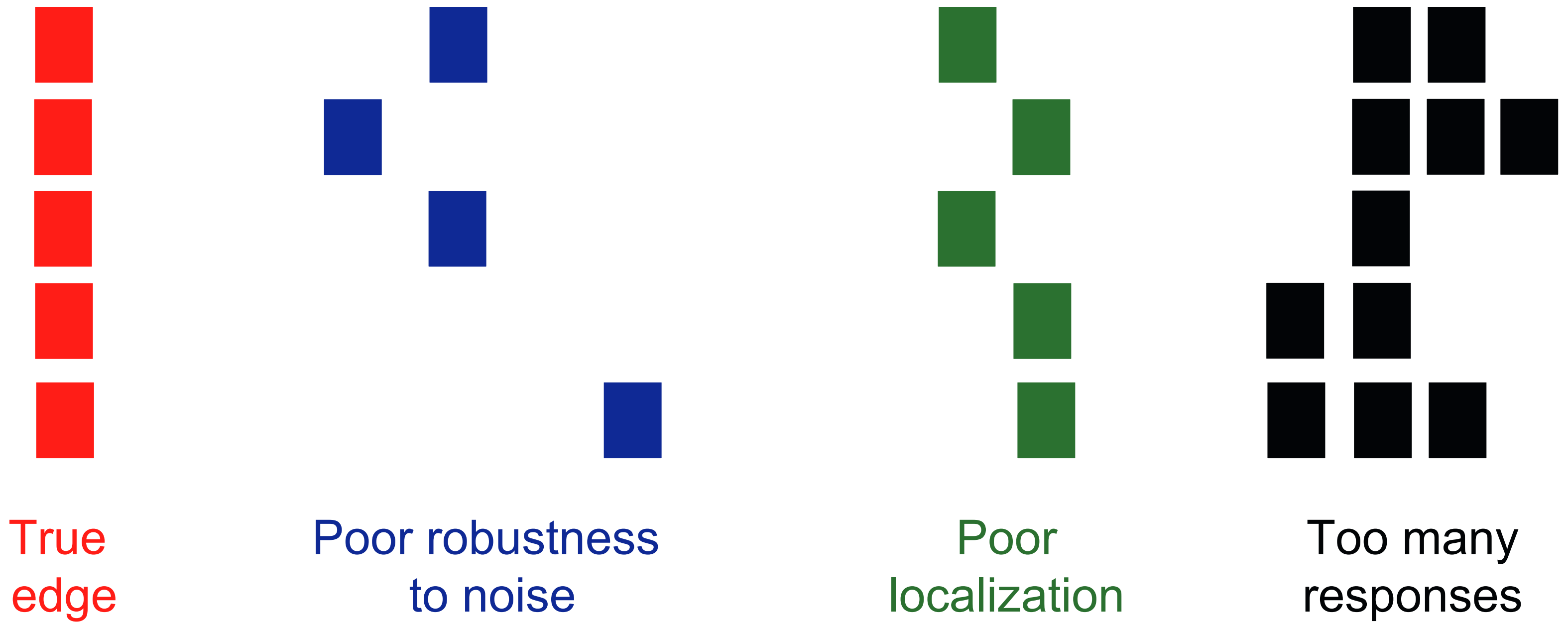
- Eliminates noise edges.
- Makes edges smoother and thicker.
- Removes fine detail.

Edge Detection

- Criteria for optimal edge detection:
  - Good detection: the optimal detector must minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges)
  - Good localization: the edges detected must be as close as possible to the true edges.
  - Single response constraint: the detector must return one point only for each true edge point; that is, minimize the number of local maxima around the true edge

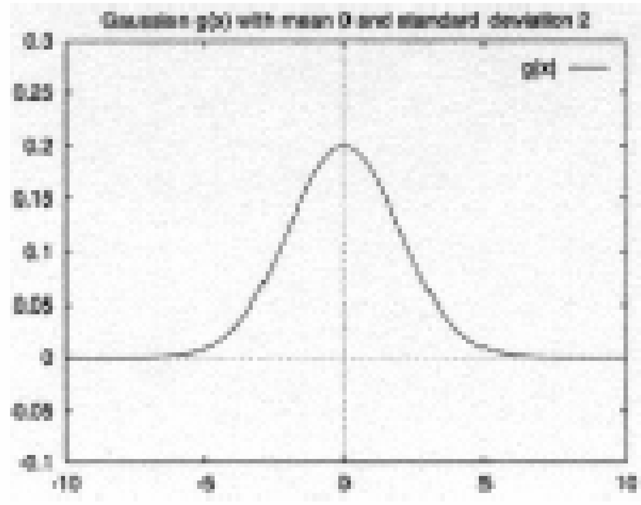
Edge Detection

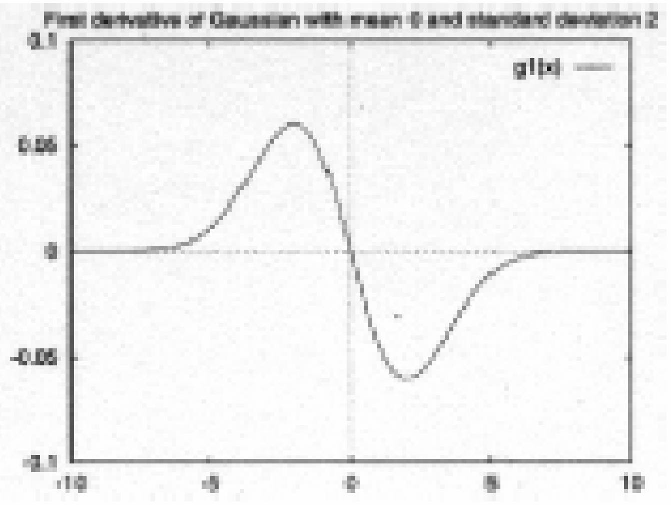
- Examples:



Canny Edge Detection

- The Canny edge detector:
  - This is probably the most widely used edge detector in computer vision.
  - Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise* ratio and localization.
  - His analysis is based on "step-edges" corrupted by "additive Gaussian noise".





Canny Edge Detection

Steps:

1. Smooth image w/ Gaussian filter
2. Apply derivative of Gaussian
3. Find magnitude and orientation of gradient
4. ‘Non-maximum suppression’
   - Thin multi-pixel wide “ridges” down to single pixel width
5. ‘Hysteresis’: Linking and thresholding
   - Low, high edge-strength thresholds
   - Accept all edges over low threshold that are connected to edge over high threshold

- Matlab: `edge(I, ‘canny’)`

Canny Edge Detector
First Two Steps

- Smoothing

$$S = I * g(x,y) = g(x,y) * I$$

$$g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

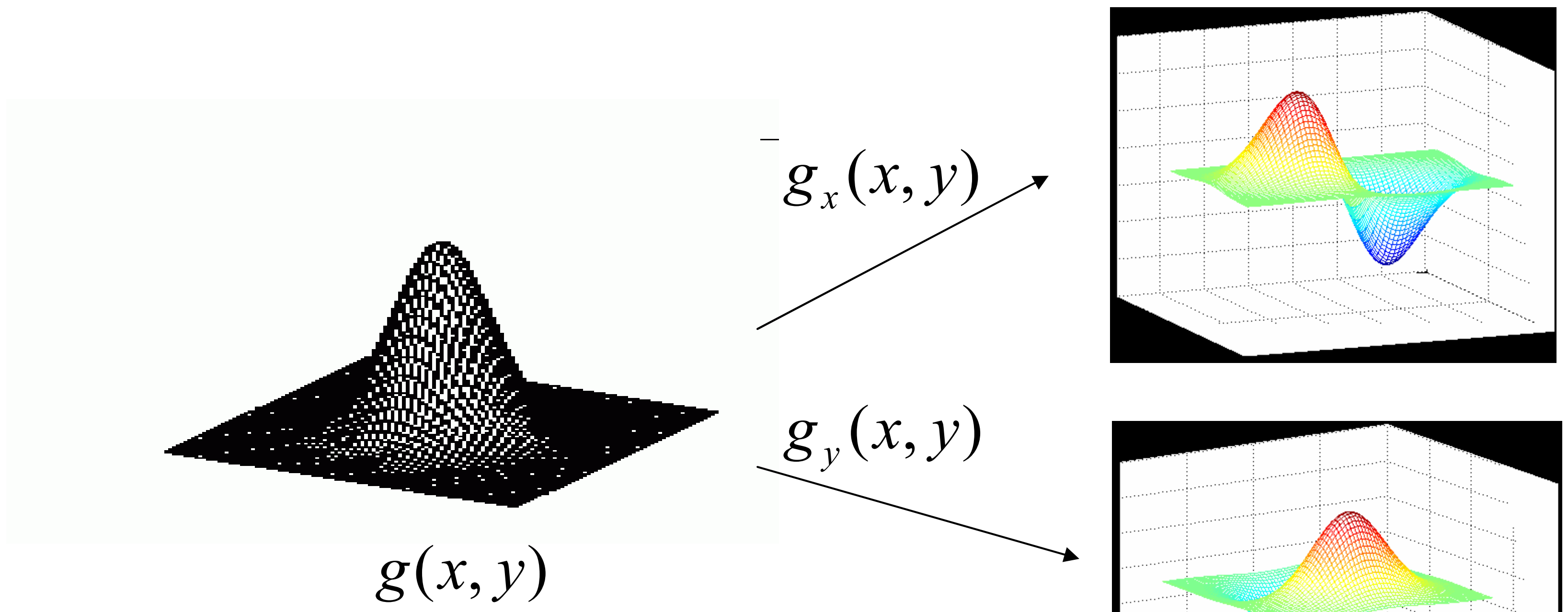
- Derivative

$$\nabla S = \nabla(g * I) = (\nabla g) * I$$

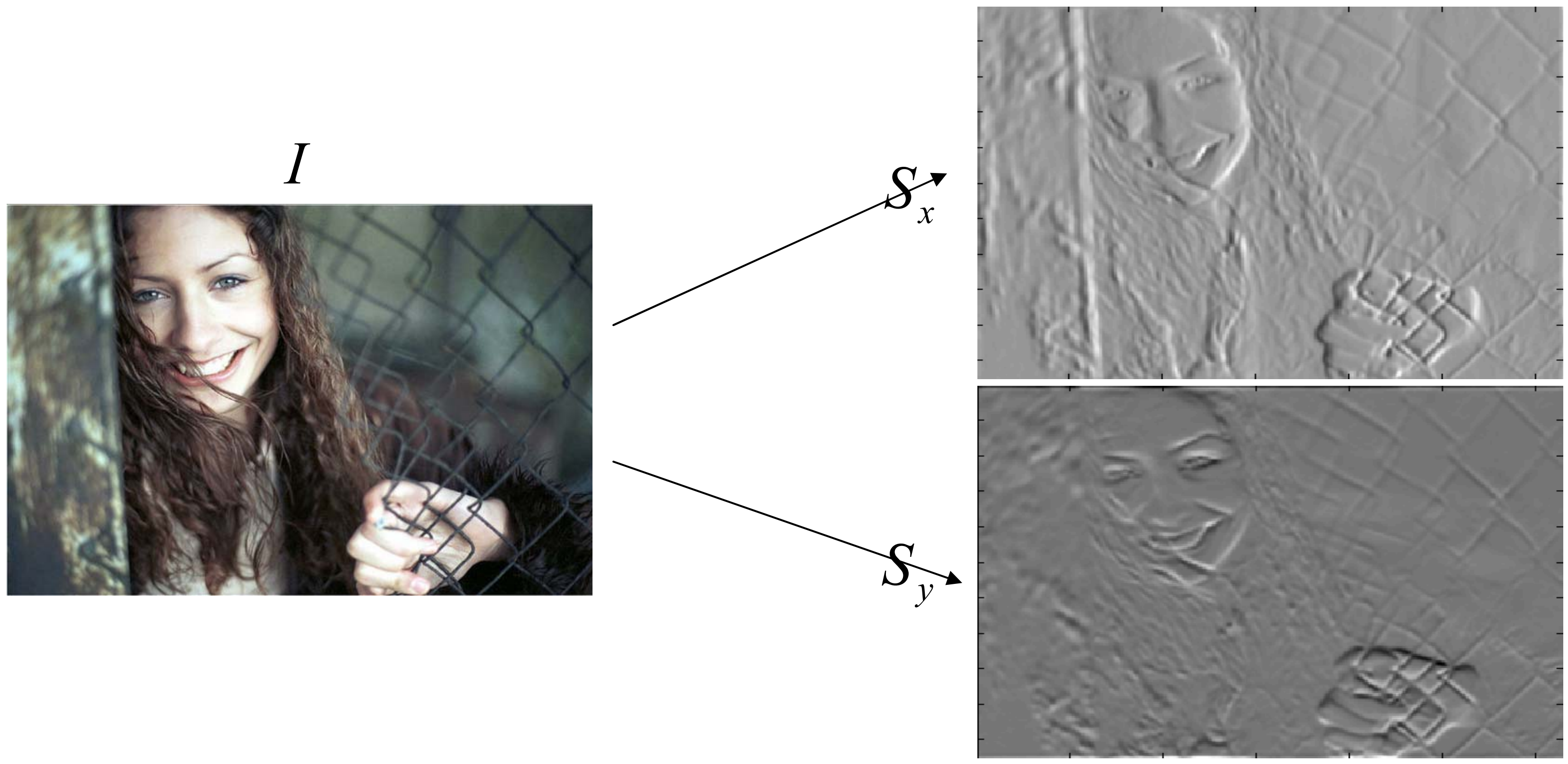
$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$

$$\nabla S = \begin{bmatrix} g_x \\ g_y \end{bmatrix} * I = \begin{bmatrix} g_x * I \\ g_y * I \end{bmatrix}$$

Canny Edge Detector
Derivative of Gaussian



Canny Edge Detector
First Two Steps



Canny Edge Detector
Third Step

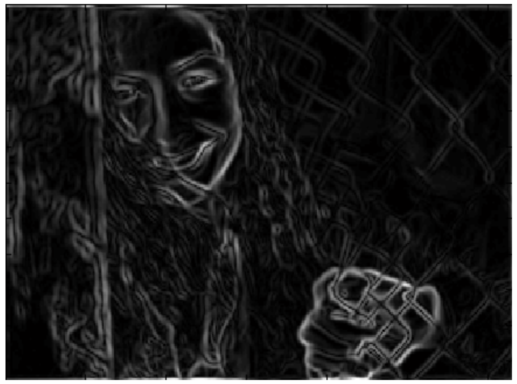
- Gradient magnitude and gradient direction

(S_x, S_y) Gradient Vector

magnitude $= \sqrt{(S_x^2 + S_y^2)}$

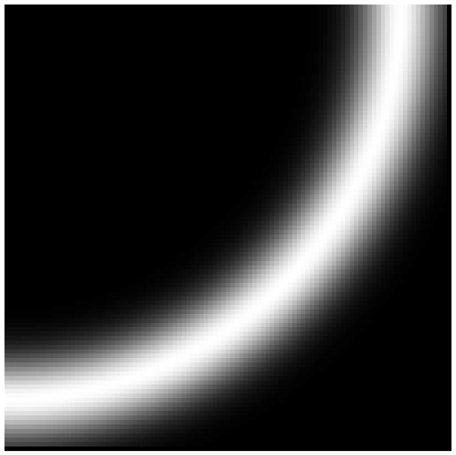
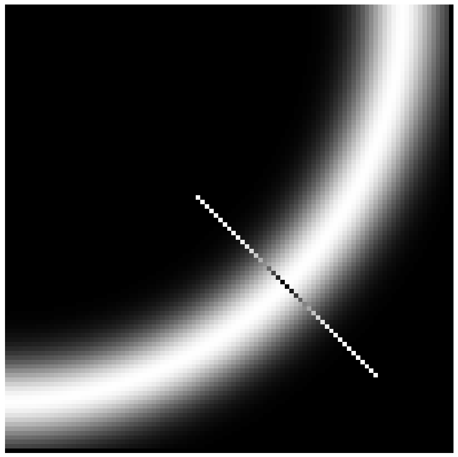
direction $= \theta = \tan^{-1} \dfrac{S_y}{S_x}$

image


gradient magnitude

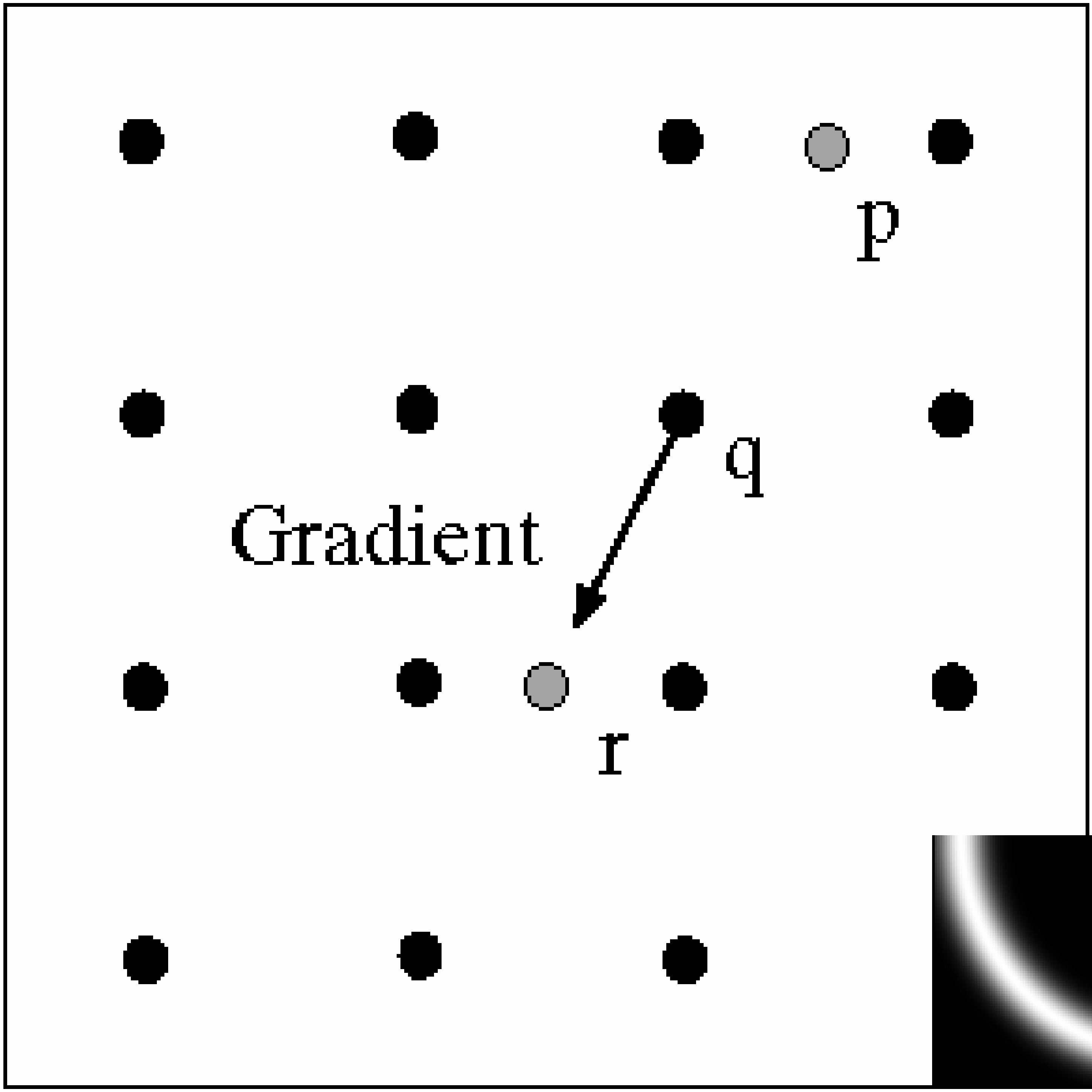
Canny Edge Detector
Fourth Step

- Non maximum suppression

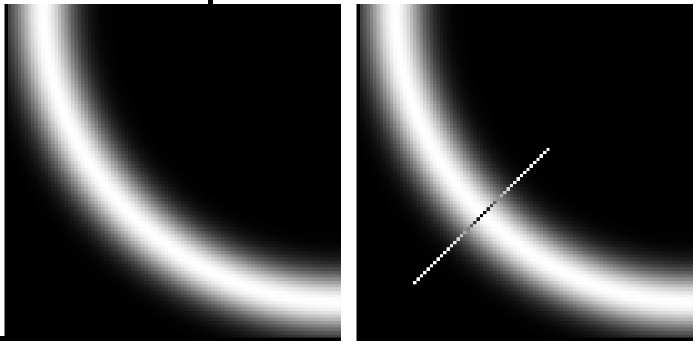



We wish to mark points along the curve where the **magnitude is biggest**. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: at which point is the maximum, and where is the next one?

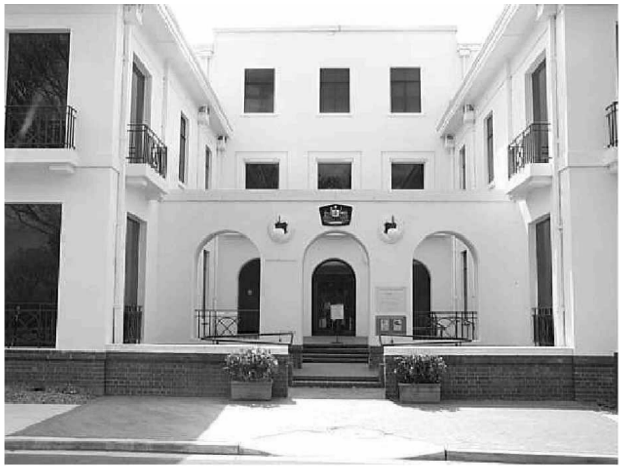
Non-maximum suppression

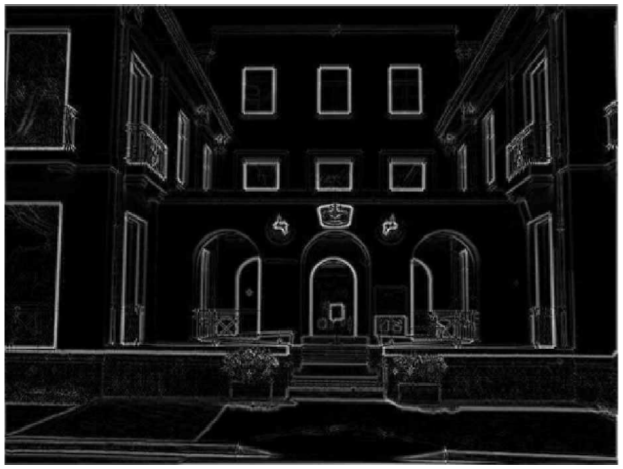


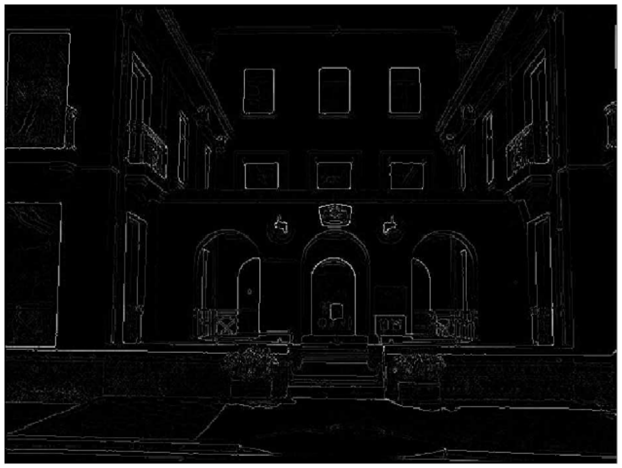
At q, the value must be larger than values interpolated at p or r.



Examples: Non-Maximum Suppression

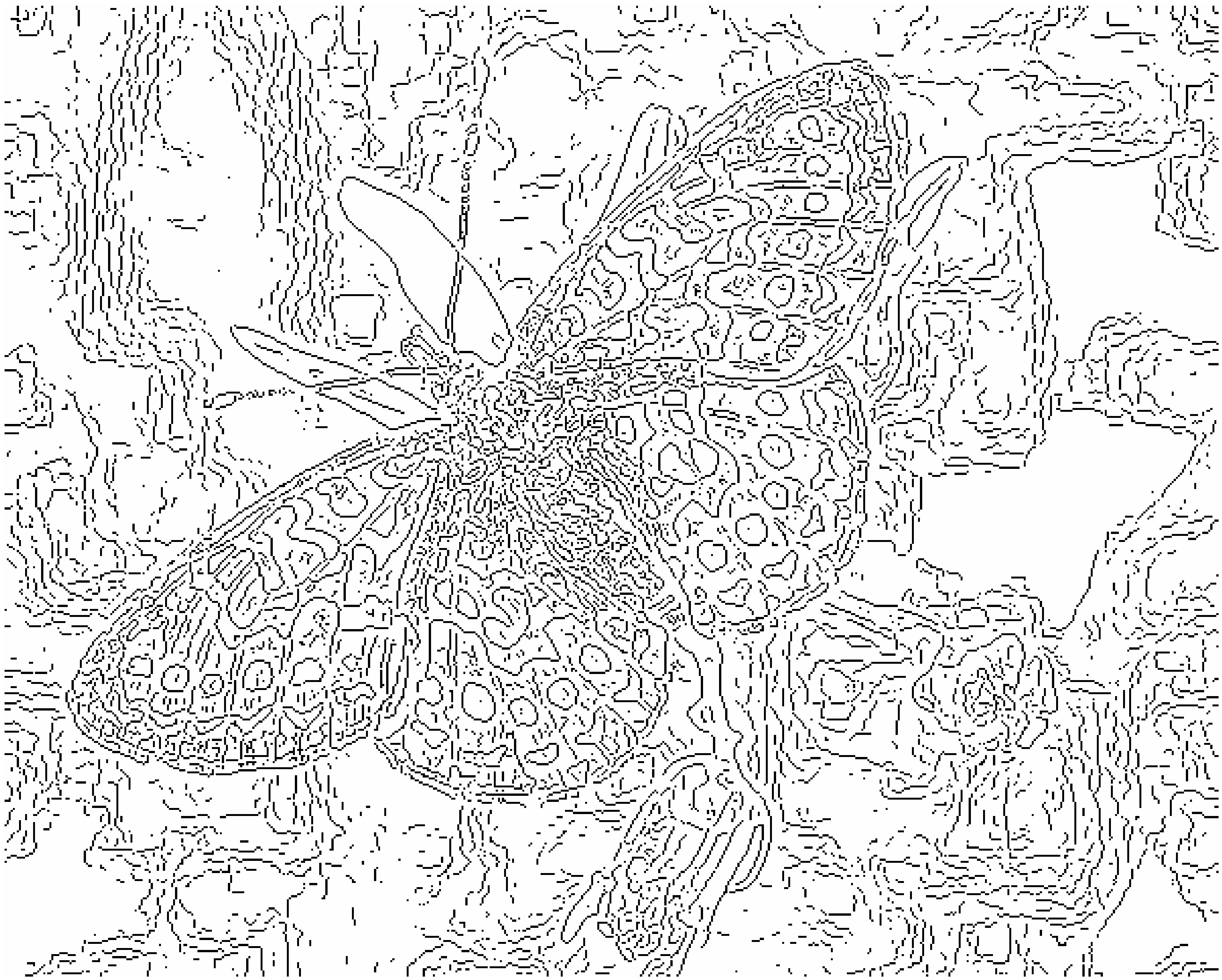

Original image


Gradient magnitude

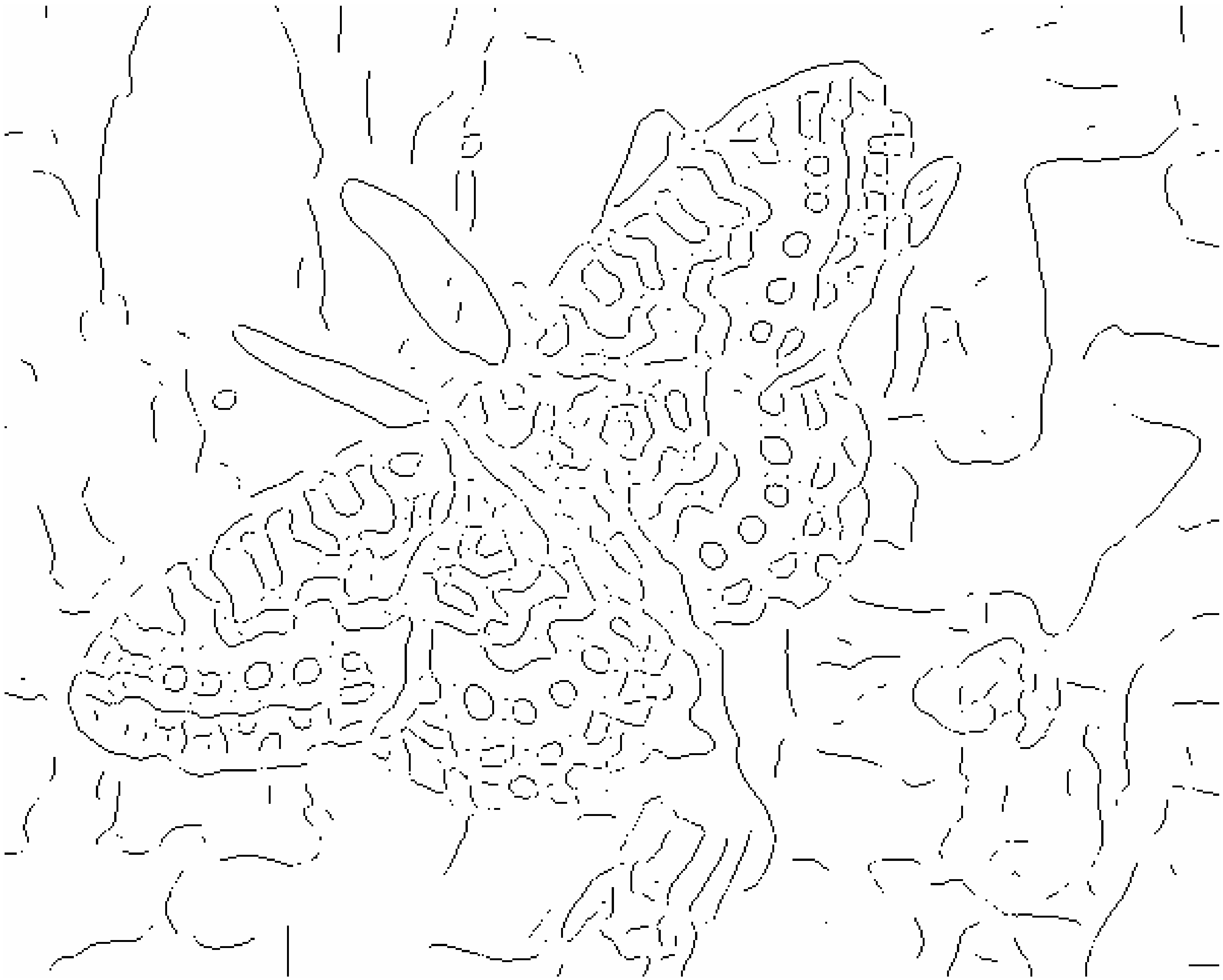

courtesy of G. Loy

Non-maxima suppressed

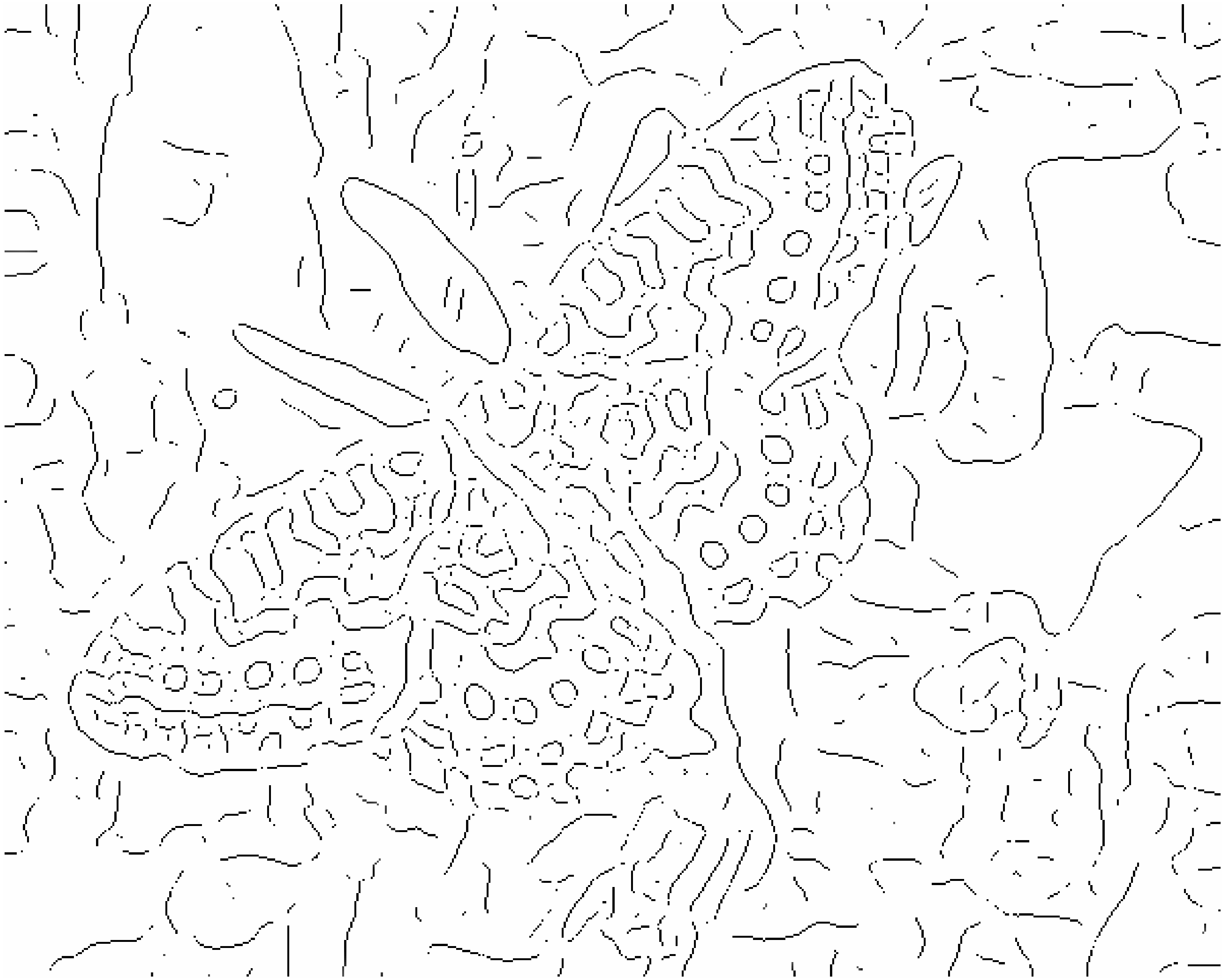
Slide credit: Christopher Rasmussen



fine scale
high
threshold

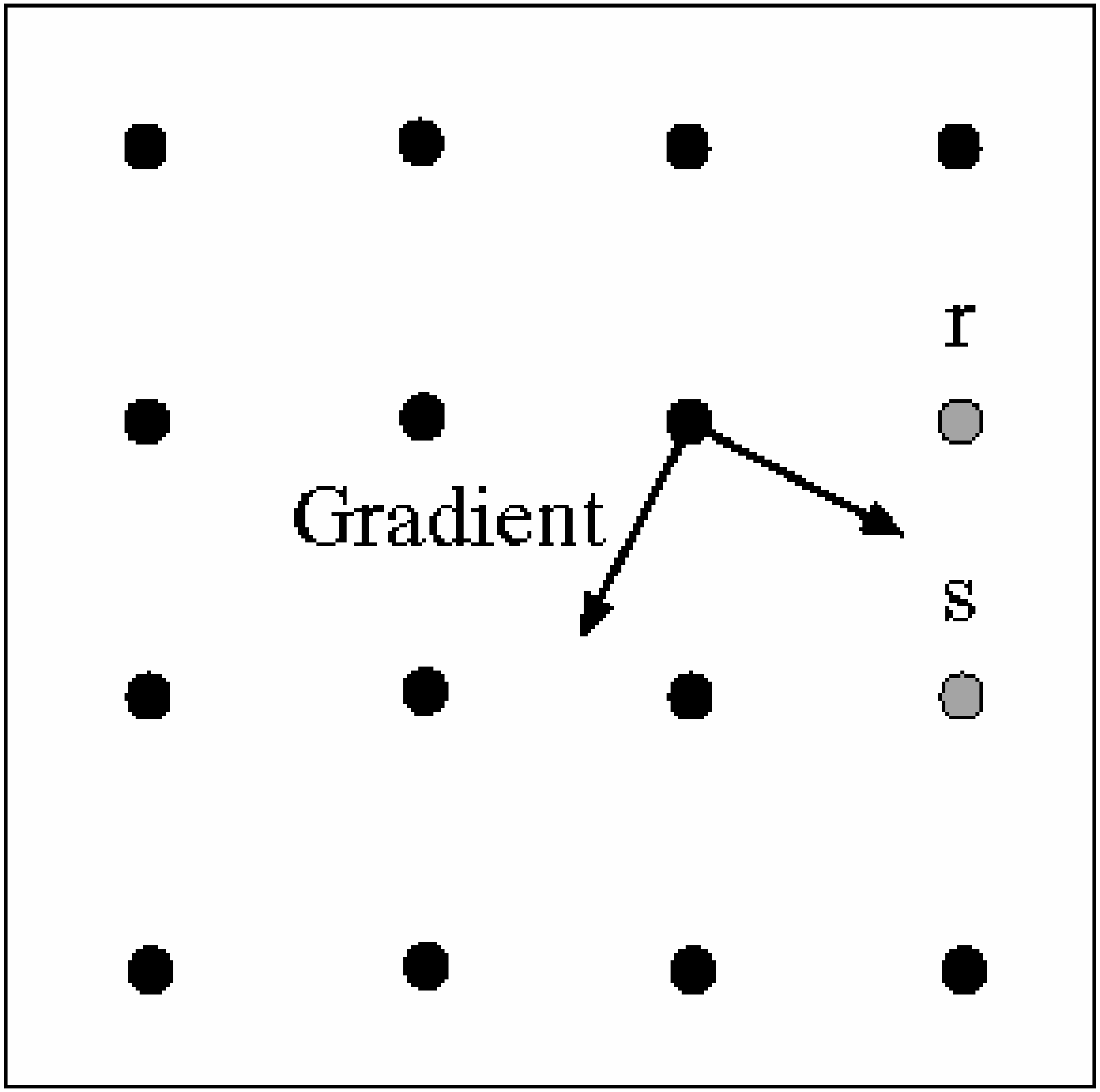


coarse scale, high threshold



coarse
scale
low
threshold

Linking to the next edge point



Assume the marked point is an edge point.

Take the normal to the gradient at that point and use this to predict continuation points (either r or s).

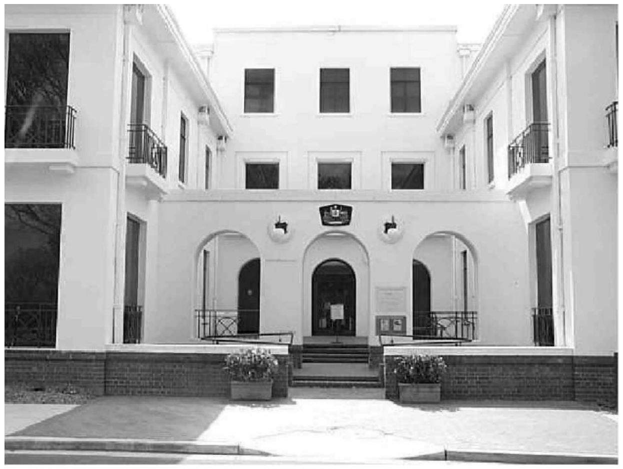
Canny Edge Detector
Step 5: Hysteresis Thresholding

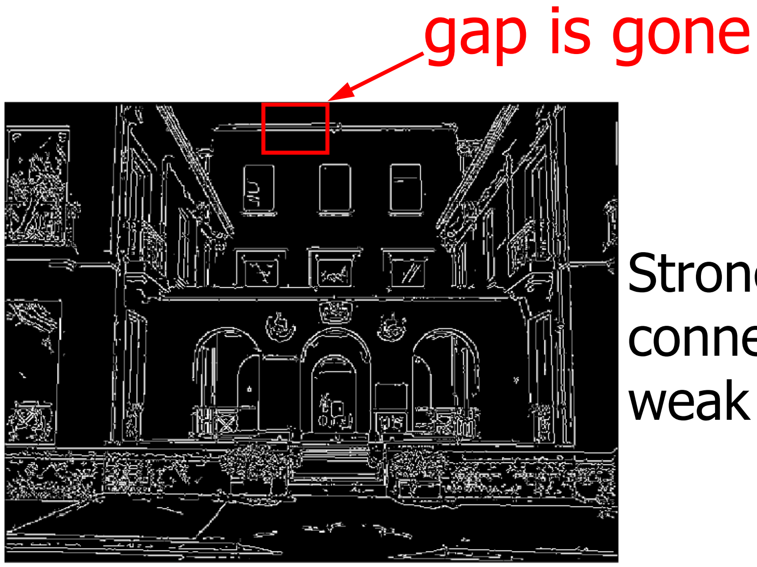
- **Hysteresis**: A lag or momentum factor
- Idea: Maintain two thresholds k_{high} and k_{low}
  - Use k_{high} to find strong edges to start edge chain
  - Use k_{low} to find weak edges which continue edge chain
- Typical ratio of thresholds is roughly

$$k_{high} / k_{low} = 2$$

Example: Canny Edge Detection

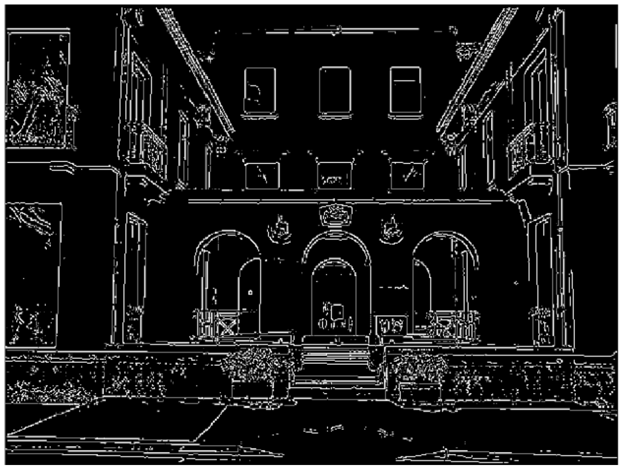
Original image

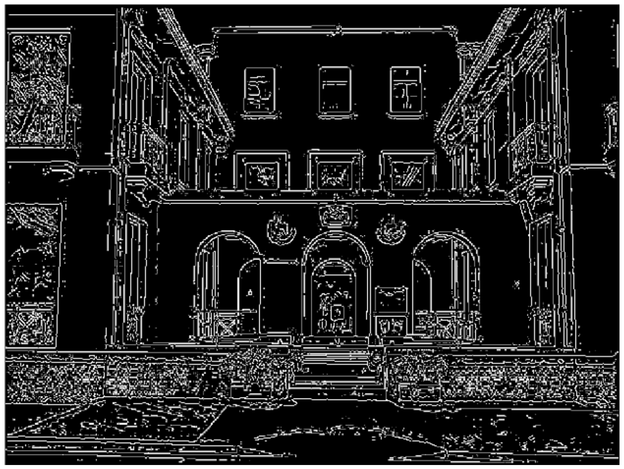




Strong + connected weak edges

Strong edges only

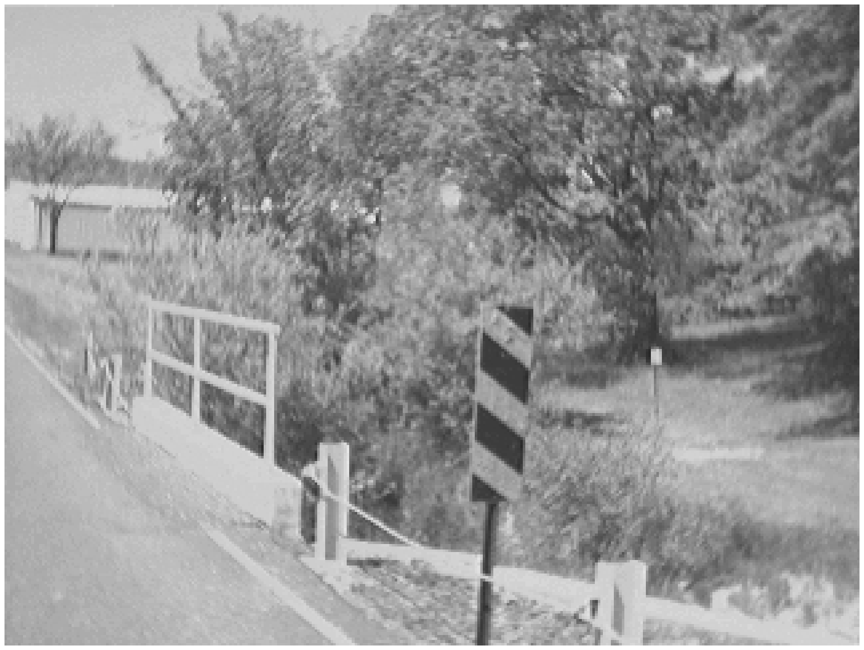
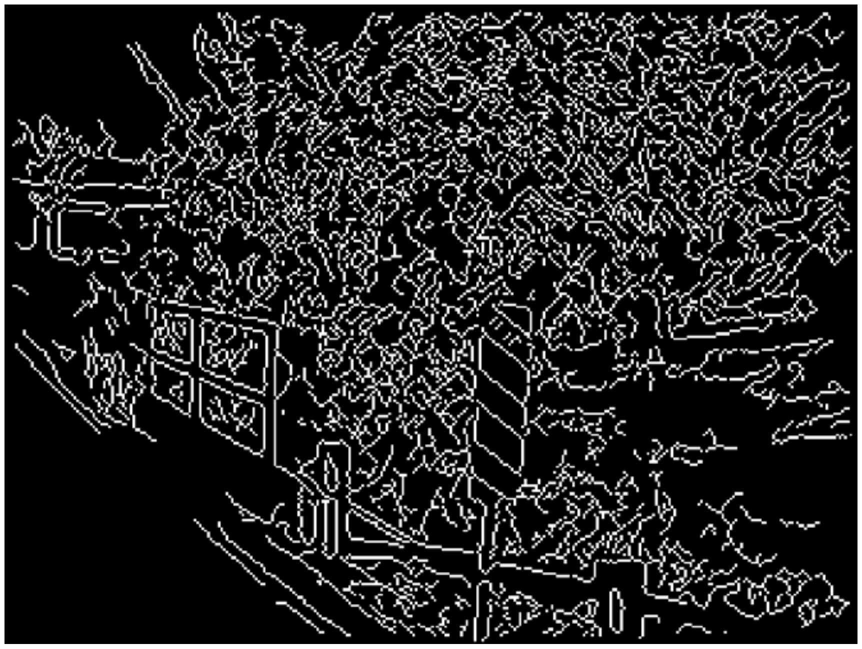




Weak edges

courtesy of G. Loy

Example:
Canny Edge Detection

Using Matlab with default thresholds

Slide credit: Christopher Rasmussen